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Strategic Deviations in Optimal Monetary Policy

Fabio Canetg

18-17

July 2018

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Schanzeneckstrasse 1
Postfach 8573
CH-3001 Bern, Switzerland
http://www.vwi.unibe.ch

Strategic Deviations in Optimal Monetary Policy*

Fabio Canetg†

Department of Economic, University of Bern

July 19, 2018

Abstract

This paper investigates the circumstances under which a central bank is more or less likely to deviate from the optimal monetary policy rule. The research questions is addressed in a simple New Keynesian dynamic stochastic general equilibrium (DSGE) model in which monetary policy deviations occur endogenously. The model solution suggests that higher future central bank credibility attenuates the current period policy trade-off between a stable inflation rate and a stable output gap. Together with the loss of credibility after a policy deviation, this provides the central bank with an incentive to implement past policy commitments. My main result shows that the central bank is willing to implement past policy commitments if a sufficient fraction of agents is not aware of the exact end date of the policy commitment. This finding challenges the time-inconsistency argument against monetary policy commitments and provides a potential explanation for the repeated implementation of monetary policy commitments in reality.

JEL classification: E42, E52, E58.

Keywords: optimal monetary policy, strategic deviations, forward guidance

*I thank Tamara Bischof, Matea Büchler, Fabrice Collard, Christian Myohl, Dirk Niepelt, Yannic Stucki, Jacqueline Thomet, Boris Thurm, Raphael Wewita, and David Weisstanner for various feedback and suggestions. Furthermore, I thank participants of the Annual Congress of the Swiss Society of Economics and Statistics in St. Gallen (2018), the RGS Doctoral Conference in Essen (2018), the BDP16 Alumni Conference in Engelberg (2018), the Gerzensee Alumni Conference in Gerzensee (2017), and the research seminars at the University of Bern for insightful comments and helpful discussions.

†University of Bern, Department of Economics, Schanzeneckstrasse 1, 3001 Bern, Switzerland, fabio.canetg@vwi.unibe.ch

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1 Introduction

Central banks have recently used more or less explicit policy commitments to manage public expectations. For example, the Swiss National Bank (SNB) promised to defend a EUR/CHF exchange rate floor with „utmost determination“ (September 6, 2011). Somewhat less explicit, the Federal Reserve Bank (Fed) „anticipate[d] (...) exceptionally low levels of the federal funds rate for some time“ (December 16, 2008). Similarly, the European Central Bank (ECB) „expecte[d] the key ECB interest rates to remain at present or lower levels for an extended period of time“ (Juli 4, 2013).

An open question is, however, under which circumstances a central bank is more or less likely to deviate from the announced policy path. Moreover, it is unclear how *future* central bank credibility interacts with the incentives to implement past policy commitments. The two research questions are: First, how does future central bank credibility affect the optimal monetary policy rule? Second, under which circumstances is it optimal for the central bank to implement past policy commitments?

This paper connects to various strands of the literature, in particular the literature on rules versus discretion, the literature on limited commitment in monetary policy, and the literature on forward guidance. The basic conceptual framework dates back to the work of Kydland and Prescott (1977) and Barro and Gordon (1983a) on rules versus discretion. Their work studies the *permanent* temptation to deviate from a policy rule that prescribes a state-independent, pre-announced inflation rate. Inflation surprises are beneficial because they reduce the natural unemployment rate towards the time-invariant efficient unemployment rate which is below the natural unemployment rate. Both Kydland and Prescott (1977) and Barro and Gordon (1983a) find that policy rules are, in general, not enforceable (i.e. time-inconsistent), unless a commitment technology is *assumed*. In an extension, Barro and Gordon (1983b) investigate enforceable policy rules when the central bank looses reputation from a policy deviation. They find that under such circumstances, policy rules may be enforceable if they are sufficiently close to the discretionary policy prescription.

The current debate on time-inconsistency of monetary policy rules is related to the central bank's optimal response to an exogenous inflation shock. Such a cost-push shock drives a *temporary* wedge between the natural output level and the efficient output level. In other words, the debate shifted away from the central bank's permanent temptation to overstimulate as in Kydland and Prescott (1977) and Barro and Gordon (1983a) towards the central bank's temporary temptation to deviate from the optimal monetary policy rule. More specifically, following Galí (2015), Woodford (2005), and Clarida et al. (1999), a central bank faces a policy trade-off in the presence of a cost-push shock: Either it stabilizes the inflation rate or it stabilizes the output gap. The optimal response to a cost-push shock is to smooth the response of the inflation rate and the output gap over time. Under full commitment, the central bank can deliver such an outcome, even though the optimal policy path may be time-inconsistent.[1] In contrast, under discretion a central bank lacks the credibility to effectively commit to a future policy path. Being constrained by that, the central bank must (sub-optimally) react more forcefully in the period when the cost-push shock hits.

Problematically, both full commitment and discretion are implausible on theoretical and empirical grounds. Concerning full commitment, it is unclear how a central bank can prevent itself from a favorable policy deviation once time passes. Monetary policy decisions are usually taken by a committee in which individual members serve for some years only. Consequently, later cohorts can overturn commitments of earlier cohorts. Discretion, on the other hand, has become a less appealing concept in light of recent monetary policy conduct ("forward guidance"): It seems implausible to assume that a central bank issues a statement regarding its future policy conduct without *any* intention to deliver. For example, Woodford (2012) argues that any form of forward guidance is in part interpreted as a policy commitment with some, but limited commitment.

As a consequence, researchers have recently started to study limited commitment in optimal monetary policy, as for example Debortoli et al. (2014), Debortoli and Lakdawala (2014), and Schaumburg and Tambalotti (2007). In their models, the central bank is *exogenously* selected to deviate from past policy commitments with a time-invariant, state-independent probability. My work suggests that such a time-invariant limited commitment scenario is not fully compatible with strategic policy decisions: In fact, central bank credibility (which is defined as the probability with which the central bank commitment is expected to be implemented in the future) is either time-varying or equal to one of the two extreme cases (full credibility or zero credibility).

[1]Barro and Gordon (1983a, 599-600) argue that it is "deceptive" to term a policy rule "time-inconsistent" when "policymakers [have] incentives to deviate from the rule when agents expect it to be followed." They claim that "the incentives to deviate from the rule are irrelevant, since commitments are assumed to be binding. Thus, the time-inconsistency of the optimal solution is (...) irrelevant when commitments are feasible." Somewhat less restrictive, Clarida et al. (1999) define time-consistency as the absence of "incentives to change its plans in an unexpected way." I will use the latter definition of time-consistency.

Some of the literature on forward guidance also considers with monetary policy commitments. Bodenstein et al. (2012), for example, define forward guidance as the explicit commitment to implement policy in accordance with the optimal monetary policy rule under time-invariant limited credibility. Similar to Debortoli et al. (2014), the timing of a policy deviation is, however, exogenous, provoking outcomes which are inconsistent with basic economic logic, namely policy deviations when the implementation of past policy commitments would have delivered a higher welfare. Such outcomes are ruled out in my model.

Haberis et al. (2014) model forward guidance as an imperfectly credible interest rate peg. They assume that the central bank's credibility increases with a (time-varying) fixed cost associated to a policy deviation. My model is more transparent about the nature of this cost: A policy deviation is costly because it is associated to higher future macroeconomics volatility, arising from the assumption that the central bank looses all its credibility forever as soon as it first deviates from the announced policy path. Furthermore, in Haberis et al. (2014), the actual decision of whether or not to implement past policy commitments is (yet again) just a coin-toss. It is hence subject to the critique that this may force the central bank to deviate even though it would have preferred to deliver.

In sum, my paper investigates the effects of future central bank credibility on the optimal monetary policy rule, as well as the circumstances under which a central bank is more or less likely to deviate from the announced policy path. It applies a simple New Keynesian dynamic stochastic general equilibrium (DSGE) model in which the central bank decides strategically whether or not to honor past policy commitments. Endogenous policy deviation come with a permanent and complete loss of central bank credibility, i.e. higher macroeconomic volatility in the future.

My results show that higher future expected central bank credibility attenuates the current period policy trade-off between a stable inflation rate and a stable output gap. This provides the central bank with an incentive to implement past policy commitments. I find that the central bank is willing to implement past policy commitments if a sufficient fraction of agents is not aware of the exact end date of the policy commitment.

The remaining of this paper is organized as follows. Section 2 derives the solution to the optimal monetary policy problem under limited credibility. Furthermore, it introduces the notion of strategic policy decisions. Section 3 presents the results of the analysis. Section 4 concludes.

2 Model

2.1 Model Environment And Optimal Monetary Policy

I analyze optimal monetary policy with strategic policy deviations and limited commitment in a simple New Keynesian dynamic stochastic general equilibrium (DSGE) model similar to Galí (2015).

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

$$C_t = \left(\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{2}$$

The representative household maximizes its utility function (equation 1) which is additively separable in a consumption bundle (C_t) and leisure ($1 - N_t$). $\beta \in (0,1)$ is the discount factor of the household, $\sigma \geq 0$ the constant relative risk aversion or, equivalently, the inverse intertemporal elasticity of substitution, and $\varphi \geq 0$ the inverse of the Frisch labor supply elasticity, i.e. the inverse wage elasticity of hours worked (N_t). A taste for variety over intermediary consumption goods C_{it} enters via the Dixit-Stiglitz (Dixit and Stiglitz (1977)) aggregator (equation 2) in which the elasticity of substitution is given by $\epsilon \in (1, \infty)$.

$$P_t C_t + Q_t B_{t+1} - W_t N_t - D_t - B_t = 0 \tag{3}$$

The household problem is subject to a sequence of dynamic budget constraints (equation 3) and a No-Ponzi condition. P_t denotes the price level, Q_t the period t price of a risk-free security B_{t+1} which pays one unit in period $t+1$, W_t the nominal wage, and D_t the (aggregated) nominal firm profit.

There is a continuum of monopolistically competitive firms on the interval $[0,1]$, each producing a single differentiated intermediary output good Y_{it} with a linear technology $Y_{it} = N_{it}$ (with N_{it} being the labor demand of firm i). As in Calvo (1983), in every period a firm decides on a new price P_{ti} of its output good with probability $1-\theta$. $Y_{t+k|t}$ is the output of a firm that last reset its price in period t, $\mathcal{C}_{t+k}(Y_{t+k|t})$ the associated nominal cost function, and $\Lambda_{t,T} \equiv \beta^{T-t} \frac{U_{C,T}}{U_{C,t}}$ the stochastic discount factor.

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right] \tag{4}$$

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \tag{5}$$

If selected to change its price, firm i maximizes the period t market value of its current and discounted future expected real profits over P_t^* (expression 4), subject to a sequence of demand constraints (equation 5). The log-linearized non-policy equilibrium of the model is given by the dynamic IS equation (6) and the New Keynesian Philipps curve (7)

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma} \left(i_t - \mathbb{E}_t \pi_{t+1} - \rho \right) \tag{6}$$

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t \tag{7}$$

where x_t is the efficient output gap, π_t the inflation rate, u_t the cost-push shock, i_t the nominal interest rate, ρ the steady state real interest rate, and $\kappa \equiv \xi(\sigma + \varphi)$ with $\xi \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.[2]

The central bank is a benevolent planer who aims at maximizing the welfare of the representative household. Borrowing from Galí (2015) and Woodford (2005), the welfare loss function is approximated by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} (\pi_t^2 + \vartheta x_t^2) \tag{8}$$

if the central bank operates under full commitment, i.e. if commitments are honored with probability 1. The weight of the output gap in the welfare loss function is given by $\vartheta \equiv \frac{\xi}{\epsilon} (\sigma + \varphi)$.[3]

Naturally, the central bank can only control the household's expectations in as far as the household anticipates the central bank to honor its commitments. This is important because the allocations off the path on which commitments are honored are exogenous to the central bank problem.

[2]Details of the derivation are provided in the appendix, section 5.1.
[3]Details of the derivation are provided in the appendix, section 5.3.

2.2 Optimal Monetary Policy under Limited Commitment

Building on the work of Debortoli and Lakdawala (2014) and Debortoli et al. (2014), who derive the welfare loss function under limited commitment, I additionally introduce time-variation in central bank credibility.[4]

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \prod_{i=0}^{t-1} \gamma_i \frac{1}{2}(\pi_t^2 + \vartheta x_t^2) \tag{9}$$

where γ_t denotes the central bank's credibility in period t and $\prod_{i=0}^{-1} \gamma_i = 1$. Note that γ_0 (rather than γ_1) is associated to (x_1, π_1) because the probability with which the household expects the period 0 commitment to be implemented in period 1 is governed by the central bank's credibility in period 0. The policy problem is subject to the New Keynesian Phillips curve

$$\pi_t = \kappa x_t + \beta\gamma_t \mathbb{E}_t \pi_{t+1} + \beta(1-\gamma_t)\mathbb{E}_t \pi_{t+1}^d + u_t \tag{10}$$

where $\mathbb{E}_t \pi_{t+1}$ is the inflation rate that is expected to prevail if commitments are honored in period $t+1$ and $\mathbb{E}_t \pi_{t+1}^d$ the inflation rate that is expected to prevail if the central bank deviates from the announced policy path in period $t+1$. Assume that the inflation rate which is expected to prevail if the central bank reneges on past policy commitment in $t+1$ is an arbitrary (linear) function of the state variable(s) in $t+1$. Formally, assume $\mathbb{E}_t \pi_{t+1}^d = \mathbb{E}_t f_{t+1}(u_{t+1})$ with the (time-varying) functional form of f_{t+1} unknown. Expressed as a Lagrangian, the central bank problem is

$$\begin{aligned} \mathcal{L} =& \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \prod_{i=0}^{t-1} \gamma_i \Bigg\{ \frac{1}{2}(\pi_t^2 + \vartheta x_t^2) + \lambda_t \Bigg(\pi_t - \kappa x_t \\ & - \beta\gamma_t \pi_{t+1} + \beta(1-\gamma_t) f_{t+1}(u_{t+1}) - u_t \Bigg) \Bigg\} \end{aligned} \tag{11}$$

with λ_t being the Lagrange multiplier associated to the New Keynesian Phillips curve in period t.[5] Combining and iterating on the first order conditions with respect to π_t and x_t yields

$$x_t = -\frac{\kappa}{\vartheta} \left[\pi_t + \pi_{t-1} + \ldots + \pi_0 - \lambda_{-1} \right] \tag{12}$$

[4] The derivation is provided in the appendix, section 5.4.

[5] The derivation of the model solution is provided in the appendix, section 5.5.

if $\gamma_i > 0 \ \forall i \in \{0, \ldots, t-1\}$.[6] By construction, deviations from the announced policy path in period t imply $\lambda_{t-1} = 0$, as in Debortoli et al. (2014).[7] Consequently, with $t = 0$ being the initial period of the policy plan

$$x_t = -\frac{\kappa}{\vartheta}\hat{p}_t \tag{13}$$

where $\hat{p}_t \equiv \pi_t + \hat{p}_{t-1}$ and $\hat{p}_{-1} = 0$.[8] For $t > 0$, the optimal output gap depends not only on the current inflation rate but also on lagged inflation rates. That is, there is a history dependence in the optimal output gap. This finding previews the result that under (limited) credibility it is both possible and optimal to commit to future policy responses when facing a current period cost-push shock. The reason is that such a commitment affects the household's expectations which in turn affect current period variables (in particular, the inflation rate). Consequently, less of a current period variability in the output gap is necessary to achieve the optimal inflation rate. This is beneficial because the welfare loss function is strictly convex in the inflation rate and the output gap. To solve the model, re-express the New Keynesian Phillips curve in terms of $\hat{p}_t$.

$$\hat{p}_t = \mu_t \left[\hat{p}_{t-1} + \beta\gamma_t \mathbb{E}_t \hat{p}_{t+1} + \beta(1-\gamma_t)\mathbb{E}_t \pi^d_{t+1} + u_t\right] \tag{14}$$

with $\mu_t \equiv \frac{\vartheta}{\vartheta(1+\beta\gamma_t)+\kappa^2}$. Suppose $u_t \sim AR(1)$ with $\mathbb{E}(\varepsilon^u_t) = 0$ and $V(\varepsilon^u_t) = \sigma^2_{\varepsilon^u_t}$ and guess the time-varying solution for $\hat{p}_t$ to be a linear function of $\hat{p}_{t-1}$ and u_t.

$$u_t = \rho_u u_{t-1} + \varepsilon^u_t \tag{15}$$

$$\hat{p}_t = a_t \hat{p}_{t-1} + c_t u_t \tag{16}$$

[6] With $\gamma_0 = 0$, the central bank's first order condition is $x_t = -\frac{\kappa}{\vartheta}\pi_t$.

[7] Suppose $\lambda_{-1} = 0$. Then, from equation 146 and 148 it follows that $x_0 = -\frac{\kappa}{\vartheta}\pi_0$ which is the the optimality condition for the period in which the policy plan is first implemented (cf. Galí (2015, 130, 135)). In other words, setting the non-physical $\lambda_{-1} = 0$ is akin to a deviation from the announced policy path in period 0.

[8] In the period of the policy implementation ($t = 0$), the *ratio* between the inflation rate and the output gap is independent of central bank credibilities. However, the *level* of the inflation rate and the output gap changes with $\{\gamma_t\}_{t=0}^T$ (cf. equation 20).

Further guess the time-varying (linear) solution $\pi_t^d = \hat{h}_t u_t$ with $\hat{h}_t$ unknown. Finally, assume that the central bank implements the discretionary solution with certainty in $T+i\ \forall i \geq 1$. Analyzing a finite period model ($T < \infty$) is useful because it allows to solve the model by backward iteration without any meaningful loss of economic substance. In particular, as of period T, the only optimal monetary policy rule consistent with rational expectations and perfect information is the discretionary optimal monetary policy rule. This is because the unique set of central bank credibility consistent with rational expectations and perfect information is $\gamma_{T+i} = 0\ \forall i \geq 0$.[9]

If a policy deviation occurs in period t, the central bank credibility is lost completely and permanently, i.e. $\gamma_{t+i} = 0\ \forall i \geq 1$. Plug the guess for $\hat{p}_t$ (equation 16) and the guess for π_t^d into the re-expressed Phillips curve (equation 14) and solve recursively for $a_t \in (0,1)$.

$$\hat{p}_t = \frac{\mu_t}{1-\mu_t\beta\gamma_t a_{t+1}}\left[\hat{p}_{t-1} + (1+\beta\gamma_t c_{t+1}\rho_u + \beta(1-\gamma_t)\hat{h}_{t+1}\rho_u)u_t\right] \tag{17}$$

$$a_t = \frac{\mu_t}{1-\mu_t\beta\gamma_t a_{t+1}}\ \ \forall\ t \tag{18}$$

Realize that a deviation from the announced policy path in t requires $\hat{p}_{t-1} = 0$ (cf. equation 13). From the guess for $\hat{p}_t$ (equation 16) we know that $\hat{p}_t^d = c_t u_t$. Furthermore, by definition, $\hat{p}_t^d = \pi_t^d$. Because I assume $\pi_t^d = \hat{h}_t u_t$ it must be that $\hat{h}_t = c_t\ \forall t$. Solve recursively for $c_t\ \forall t \in \{0,\ldots,T\}$, using $\{a_t\}_{t=0}^{T+1}$ from above.

$$c_t = \frac{\mu_t(1+\beta c_{t+1}\rho_u)}{1-\mu_t\beta\gamma_t a_{t+1}}\ \ \forall t \in \{0,\ldots,T\} \tag{19}$$

with $c_{T+i} = \frac{\vartheta}{\kappa^2+\vartheta(1-\beta\rho_u)}\ \forall i \geq 1$ as in Galí (2015, 130). The optimality condition (equation 13), together with the guess for $\hat{p}_t$ (equation 16) and the solution for the coefficients (in particular, $c_t = \hat{h}_t$) yields the time-varying model solution for $\gamma_t \in (0,1)$

$$x_t = a_t x_{t-1} - \frac{\hat{h}_t\kappa}{\vartheta}u_t \tag{20}$$

[9] So far, γ_t is not *constrained* to be consistent with rational expectations and perfect information. In other words, $\gamma_T > 0$ is, in principle, possible even though the central bank implements the discretionary solution in period $T+1$ with certainty.

where $\mu_t \equiv \frac{\vartheta}{\vartheta(1+\beta\gamma_t)+\kappa^2}$ $\forall t$, $a_t = \frac{\mu_t}{1-\mu_t\beta\gamma_t a_{t+1}}$ $\forall t$, $\hat{h}_t = \frac{\mu_t(1+\beta\hat{h}_{t+1}\rho_u)}{1-\mu_t\beta\gamma_t a_{t+1}}$ $\forall t \in \{0,\dots,T\}$, and $\hat{h}_{T+i} = \frac{\vartheta}{\kappa^2+\vartheta(1-\beta\rho_u)}$ $\forall i \geq 1$.

The output gap x_t depends on the *entire* sequence of current and future central bank credibilities $\{\gamma_j\}_{j=t}^T$. More specifically, $\{\gamma_j\}_{j=t}^T$ determines the optimal persistence in the output gap as well as the severity of the policy trade-off.

As a result of the classic policy trade-off, the central bank optimally commits to a conditional (future) deflation in response to a positive cost-push shock. Crucially, to decrease $\mathbb{E}_t\pi_{t+1}$ sufficiently, the central bank must announce a more pronounced deflation, the shorter the horizon over which the central bank is expected to implement the policy commitment. In other words, the central bank must implement a more persistent (negative) output gap, the sooner the central bank is expected to return to a discretionary mode. Formally, the lower T and/or the lower the values in $\{\gamma_j\}_{j=t}^T$, the higher a_t.

The degree to which the household's expectations adjust to policy commitments determines the severity of the policy trade-off between the output gap and the inflation rate in period t. More specifically, if the (representative) household expect the policy commitment to be implemented over a shorter horizon, the policy trade-off becomes more severe ($\hat{h}_t$ rises).

To illustrate, assume that the central bank's optimal policy is a commitment to a (conditional) future deflation. Since the inflation rate can be expressed as a (positive) function of discounted future expected output gaps, $\mathbb{E}_t\pi_{t+1}$ is *ceteris paribus* higher, the lower T and/or the lower the values in $\{\gamma_j\}_{j=t}^T$. From above, we know that this off-equilibrium increase in $\mathbb{E}_t\pi_{t+1}$ induces the central bank to commit to a higher persistence in the output gap. However, households discount future expected inflation rates and incur convex losses from x_t and π_t. For this reason, the (off-equilibrium) rise in $\mathbb{E}_t\pi_{t+1}$ cannot be offset completely by the central bank's optimal commitment to a higher persistence in the output gap. It is for that reason that the current period policy trade-off accentuates. Formally, the lower T and/or the lower the values in $\{\gamma_j\}_{j=t}^T$, the higher the impact coefficient ($\hat{h}_t$).

2.3 Strategic Deviations in Optimal Monetary Policy

In contrast to previous work on limited commitment in optimal monetary policy, I allow the central bank to take *strategic* policy decisions. More specifically, the central bank can either honor past policy commitments or deviate. It delivers on past policy commitments if and only if the value of doing is strictly greater than the value associated to a policy deviation.

Introducing strategic policy deviations is important for two reasons. First, it shows under which circumstances a central bank is *more or less* likely to deviate from the announced policy path. Debortoli et al. (2014) were the first who adressed this question: They report the potential welfare gains of a policy deviation over the horizon of the impulse response function to a cost-push shock. My work complements their analysis by showing that the temptation to deviate is not only time-dependent, but also state-dependent.

Second, the introduction of strategic policy deviations provides an endogenous criterion based on which we can assess *whether or not* the central bank would deviate from past policy commitments. This debate seemed to be resolved because the static perspective suggests that it is always weakly preferable to implement the discretionary solution. My work shows that there are dynamic considerations which induce the central bank to implement past policy commitments. More specifically, if for some reason it is optimal to implement policy commitments in the future, it may also become optimal to honor policy commitments today.

The strategic policy problem is a recursive representation of the central bank optimization problem (equation 9) that takes into account that the continuation values differ depending on the central bank's policy choice. Importantly, it is assumed that the central bank looses all its credibility forever as soon as it first deviates from the announced policy path. Proposition 2 proves that the complete and permanent loss of credibility implies policy deviations in every future period. In contrast, if the central bank honors past policy commitments in t, it can (again) take a strategic policy decision in $t+1$. Formally,

$$V_t^d(u_t) = \max_{\{x_{t+s},\pi_{t+s}\}_{s=0}^{\infty}} U_t^d + \beta \mathbb{E}_t V_{t+1}^d(u_{t+1}) \tag{21}$$

$$V_t^h(x_{t-1},\gamma_{t-1},u_t) = \max_{\{x_{t+s},\pi_{t+s}\}_{s=0}^{\infty}} U_t^h + \beta \mathbb{E}_t V_{t+1}(x_t,\gamma_t,u_{t+1}) \tag{22}$$

$$V_t(x_{t-1},\gamma_{t-1},u_t) = \max\left\{V_t^d(u_t), V_t^h(x_{t-1},\gamma_{t-1},u_t)\right\} \tag{23}$$

$$U_t^i = -\frac{1}{2}(\pi_{t,i}^2 + \vartheta x_{t,i}^2) \tag{24}$$

where V_t^d denotes the value associated to a policy deviation in period t, V_t^h the value associated to honored commitments in period t, and U_t^i the period objective function of the central bank evaluated at the optimal $x_{t,i}$ and $\pi_{t,i}$, i.e. evaluated at the $x_{t,i}$ and $\pi_{t,i}$ which satisfy the optimal monetary policy rule under limited credibility (equation 20 and equation 150). $x_{t,i}$ and $\pi_{t,i}$ depend on the central bank's policy choice $i \in \{d,h\}$ where d stands for a policy deviation and h stands for the implementation of past policy commitments.

2.4 The Driving Process and Model Calibration

The driving force is a cost-push shock u_t^j which evolves according to a 2-state Markov process where the (discrete) magnitude of u_t^j is indexed by $j \in \{L, H\}$. Formally,

$$\begin{bmatrix} u_t^H \\ u_t^L \end{bmatrix} = \begin{bmatrix} p_{H,H} & 1 - p_{L,L} \\ 1 - p_{H,H} & p_{L,L} \end{bmatrix} \begin{bmatrix} u_{t-1}^H \\ u_{t-1}^L \end{bmatrix} \tag{25}$$

where $1 - p_{L,L}$ denotes the probability of transitioning from the low state L to the high state H (which is associated to u_t^H). Let $u_t^H = -u_t^L$ with $u_t^H > u_t^L$ and assume, for simplicity, that $p_{H,H} = p_{L,L} = 0.5$.

The model is calibrated to quarterly data as suggested in Galí (2015, 67). In particular, $\beta = 0.99$ (implying a annualized steady state real interest rate of approximately 4%), $\epsilon = 9$ (implying a steady state mark-up of 12.5%), $\alpha = 0$ (reflecting a simplifying constant returns to scale assumption), $\sigma = 1$ (log-utility), $\psi = 5$ (implying a Frisch elasticity of the labor supply equal to 0.2), $\theta = 0.75$ (implying an average duration of a price equal to four quarters), $\rho_u = 0$ (where not stated otherwise), and $u_t^H = 0.005$.

2.5 A Simple Model With Strategic Policy Deviations

Suppose $t = \{0, 1, 2, 3\}$, with $T = 2$, and assume $x_{-1} = 0$. The central bank decides strategically whether or not to deviate from the optimal monetary policy rule in $t = \{0, 1, 2\}$. In period $T + 1$, the central bank implements the discretionary solution with certainty.

Consistency with rational expectations requires that the agents' beliefs about the number of states in which a policy deviation occurs coincide with the actual number of states in which a policy deviation occurs. For example, $\gamma_t = 1$ is consistent with rational expectations and perfect information if and only if the central bank implements past policy commitments in period $t + 1$ independent of the realization of u_{t+1}. $\gamma_t = 0.5$ is consistent if and only if past policy commitments are implemented in exactly one state in period $t + 1$ (provided that u_{t+1} can only take on two values), and $\gamma_t = 0$ is consistent if the central bank reneges on past policy commitments in period $t + 1$ independent of the realization of u_{t+1}.

Future central bank credibility affects the optimal allocation in period t via two channels: Directly via the optimal future allocation (i.e. via $\mathbb{E}_t \pi_{t+1}$) and indirectly via today's solution coefficients (a_t and h_t). For this reason, we must assess the consistency of each γ_i conditional on an entire *sequence* of

$\{\gamma_t\}_{t=i}^T$ rather than conditional on γ_i only. A consistent sequence of $\{\gamma_t\}_{t=0}^T$ is a sequence in which every individual γ_t is consistent.

3 Results

In response to a positive cost-push shock, the central bank cannot simultaneously stabilize the inflation rate and the output gap. Because the central bank objective function is convex in π_t and x_t, it is moreover suboptimal to *either* stabilize the inflation rate *or* the output gap. Consequently, the optimal response to a positive cost-push shock consists of a positive inflation rate and a negative output gap. The implementation of a negative output gap exerts negative pressure on the inflation rate and partly offsets the (off-equilibrium) rise in the inflation rate caused by the positive cost-push shock. Furthermore, the optimal policy path involves a commitment to a prolonged (conditional) recession which is accompanied by a negative inflation rate.

3.1 Future Central Bank Credibility and Current Period Allocation

The optimal monetary policy rule with strategic policy deviations shows that the persistence in the output gap (a_t) as well as the severity of the policy trade-off ($\hat{h}_t$) depend on current and future central bank credibilities (γ_{t+i} $\forall i \geq 0$). These central bank credibilities may be time-varying. My model is hence flexible enough to study the effect of future (time-varying) central bank credibilities on the current optimal monetary policy rule. By that, it advances on the research of Galí (2015), Woodford (2005) and Debortoli et al. (2014) who implicitly assume a time-invariant optimal monetary policy rule.

To illustrate the dependence of the optimal monetary policy rule on future central bank credibilities, consider two simulations (indexed by j) with a deterministic sequence of central bank credibilities $\{\gamma_t^j\}_{t=0}^{T+1}$ (where $T = 2$) and a deterministic sequence of cost-push shocks $\{u_t\}_{t=0}^{T+1} = \{H, 0, 0, 0\}$. In simulation a, the central bank credibilities are $\{\gamma_t^a\}_{t=0}^{T+1} = \{1, 1, 1, 0\}$ and in simulation b they are $\{\gamma_t^b\}_{t=0}^{T+1} = \{1, 1, 0, 0\}$. Policy commitments are assumed to be honored in $t \in \{0, 1, 2\}$ but not in $t = 3$.[10]

How does future central bank credibility affect the optimal monetary policy rule? Let us first investigate how the optimal persistence in the output gap

[10] $\gamma_t = 0$ does not *per se* imply a policy deviation in period t. It only reflects the central bank's inability to credibly commit to a future policy path. In other words, $\gamma_t = 0$ does not, *per se*, refrain the central bank from implementing past policy commitments. Similarly, $\gamma_{t-1} = 1$ does not *per se* imply honored commitments in period t because γ_{t-1} solely reflects the probability with which the agents expect future central bank commitments to be implemented.

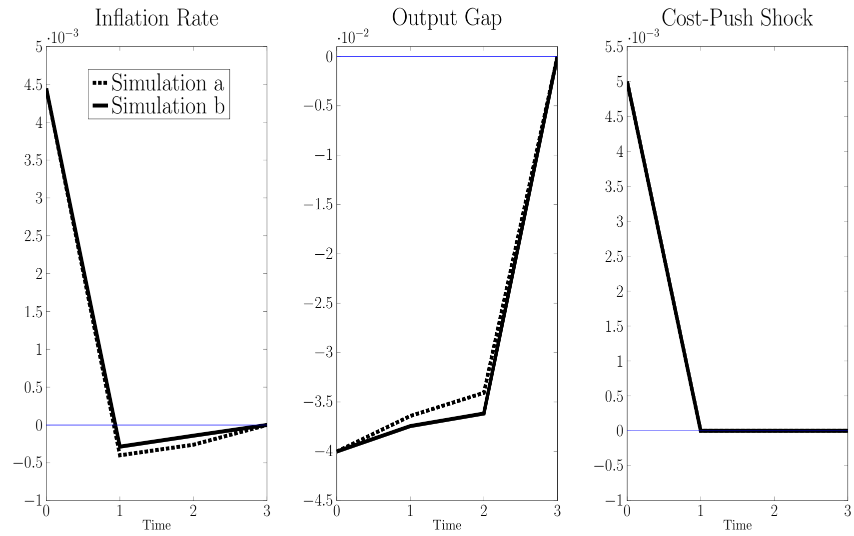

Figure 1: The Optimal Persistence ($\hat{h}_t^a = \hat{h}_t^b$)

is affected by future central bank credibilities. Suppose, for the sake of the argument, that the impact coefficient of the cost-push shock ($\hat{h}_t$) is equal in both simulations. Under this assumption, the allocation in period t depends on future central bank credibilities only because future central bank credibilites affect the optimal persistence in the output gap.

Monetary policy commitments affect the agents' expectations about future output gaps. By that, they exert (positive or negative) pressure on the current inflation rate. To see this, re-express the Phillips curve as follows:

$$\pi_0^a = \kappa \mathbb{E}_0 \sum_{i=0}^{3} \beta^i x_i^a + u_t \tag{26}$$

$$\pi_0^b = \kappa \mathbb{E}_0 \sum_{i=0}^{2} \beta^i x_i^b + u_t \tag{27}$$

In simulation a ($\gamma_2 = 1$), agents expect the central bank to honor policy commitments in period 3 while in simulation b ($\gamma_2 = 0$), agents expect a return to the feasible $(x_t, \pi_t) = (0, 0)$ allocation in period 3. Facing a positive cost-push shock in $t = 0$, a naïve central banker may consider the same policy commitment independent of future central bank credibilities. Is this optimal, knowing that $\mathbb{E}_0 x_3^a < 0$ and $\mathbb{E}_0 x_3^b = 0$? More formally, is $x_0^a = x_0^{b,G}$, $x_1^a = x_1^{b,G}$, and $x_2^a = x_2^{b,G}$ (where G denotes a guess) an optimal monetary policy response, even though $\mathbb{E}_0 x_3^a < 0$ and $\mathbb{E}_0 x_3^b = 0$?

The answer is no, because optimality would require the allocation (x_0^a, π_0^a) to deliver the same welfare as $(x_0^{b,G}, \pi_0^{b,G})$. Strict convexity in the central bank objective function suggests, however, that the two allocations do not deliver the same welfare. This is because the output gap is the same in both allocations ($x_0^a = x_0^{b,G}$) while the corresponding inflation rate is not ($\pi_0^a < \pi_0^{b,G}$). The (off-equilibrium) difference $\pi_0^a < \pi_0^{b,G}$ is *partly* offset by a commitment to a more severe recession in simulation b.

Figure 1 displays the optimal monetary policy responses for both simulations under the assumption $\hat{h}_t^a = \hat{h}_t^b \ \forall t$.[11] The output gap and the inflation rate are the same in period 0 because $x_{-1} = 0$ and because $\hat{h}_0$ is independent γ_2 by assumption. As of period 1, however, simulation b features a more negative optimal output gap than simulation a. This is because in simulation b, the central bank must optimally implement a more pronounced recession to partly offset the shorter horizon for which it can provide a credible policy commitment. The persistence parameter in the optimal monetary policy rule is hence higher in simulation b ($a_t^b > a_t^a \ \forall t \leq T$).

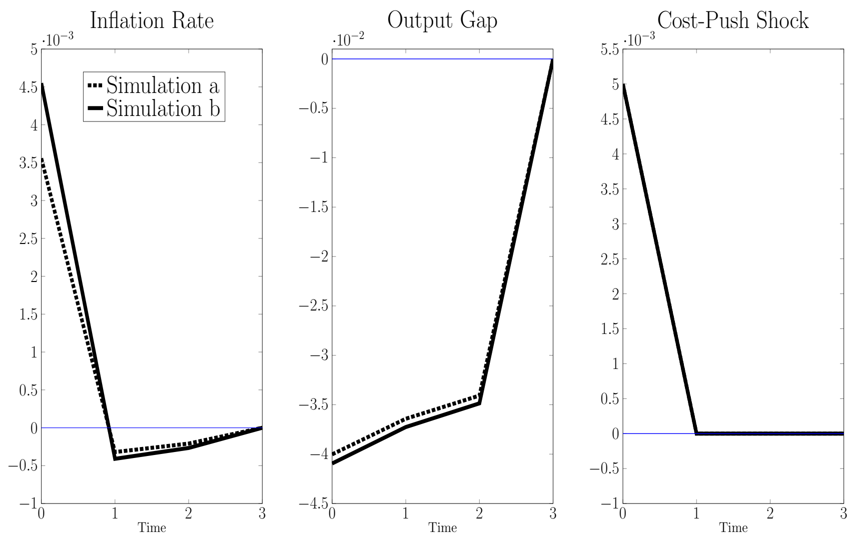

Figure 2: The Optimal Impact Coefficient ($a_t^a = a_t^b$)

Let us now analyze how future central bank credibilities affect the severity of the policy trade-off between the output gap and the inflation rate (which is formally captured by $\hat{h}_t$). To isolate this channel, suppose that a_t is left unaffected by the sequence of current and future central bank credibilities.

The severity of the policy trade-off is greater, the less flexible public expectations adjust to monetary policy commitments. As before, agents expect the policy commitment to be implemented for two (three) periods in simulation a (b). Applying the same logic as in the argument above, this implies that the optimal x_0^b must lie below x_0^a to partly offset the shorter horizon over which

[11] To see any meaningful difference between simulation a and b, figure 1 and figure 2 assume an unrealistically high degree of price stickiness ($\theta = 0.9792$). The qualitative result is, however, *not* affected by the degree of price stickiness.

the public expects the central bank to implement the policy commitments in simulation b. Importantly, the optimal x_0^b is *not* so low as to have $\pi_t^b = \pi_t^a$. The policy trade-off is hence more pronounced in simulation b.

Figure 2 illustrates the optimal monetary policy responses for both simulations under the assumption $a_t^a = a_t^b \ \forall t$. In $t = \{0, 1, 2\}$, the inflation gap and the output gap are both further off steady state than in simulation b.

Figure 3 summarizes the evolution of the optimal persistence and the optimal impact coefficient over time for both simulations. Independent of t, both coefficients are more favorable (that is: lower) in simulation a, compared to simulation b. Also, both coefficients become less favorable (that is: rise) over time because the horizon over which the central bank can provide a credible policy commitment becomes shorter, the more time has passed.

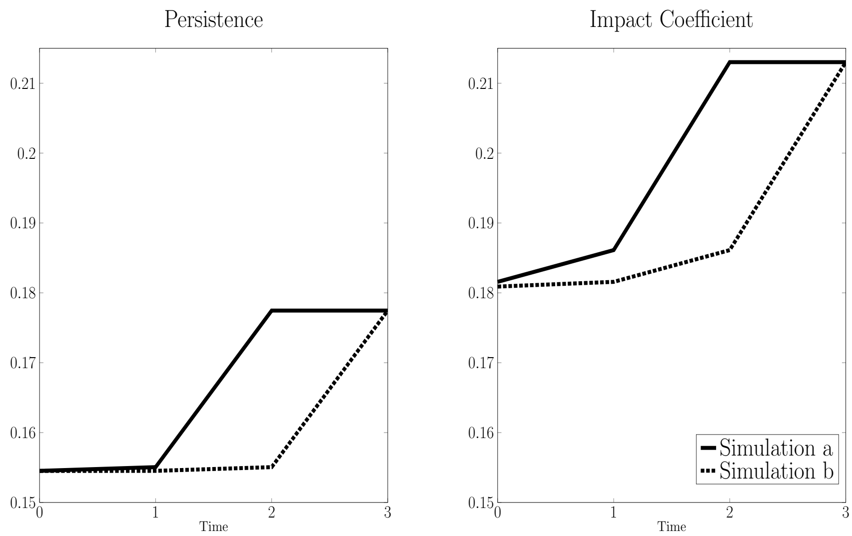

Figure 3: Optimal Monetary Policy Rule with $\rho_u = 0.95$

3.2 Policy Deviations vs. Honored Commitments

When is it optimal for the central bank to implement past policy commitments? Proposition 1 shows that if each agent forms a correct belief about the central bank's credibility in period T (which is $\gamma_T = 0$ because the central bank implements the discretionary solution in period $T+1$ with certainty) full credibility in $t < T$ is inconsistent with strategic policy decisions.

Proposition 1 *Time-invariant full credibility ($\gamma_t = 1 \ \forall t < T$) is inconsistent with strategic policy deviations.*

Proposition 1 can be proven directly: Time-invariant full credibility is inconsistent with strategic policy deviations if the value function under a policy deviation is weakly greater than the value function under honored commitments in at least one period.[12] $V_2^h(u_2^k) > V_2^d(u_2^k)$ is satisfied if and only if

$$(1-a_2)^2 + \frac{\kappa^2}{\vartheta} a_2^2 < 0 \tag{28}$$

where ϑ is strictly positive. Full credibility in period $t < T$ is inconsistent with strategic policy deviations because condition 28 cannot hold with $\gamma_T = 0$.

Proposition 2 establishes the pair result that zero credibility is consistent with strategic policy deviations if each agent forms a correct belief about the central bank's credibility in period T.[13]

Proposition 2 *Time-invariant zero credibility ($\gamma_t = 0\ \forall t \leq T$) is consistent with strategic policy deviations.*

If the central bank cannot affect the agents' expectations in period T (because all of them hold the correct belief $\gamma_T = 0$), it cannot do better than the discretionary solution. As a consequence, agents assign a zero credibility to any central bank commitment made in period $T-1$ ($\gamma_{T-1} = 0$). This, in turn, makes it optimal for the central bank to implement the discretionary solution in $T-1$, such that $\gamma_{T-2} = 0$, and so on.

Taken together, proposition 1 and 2 confirm the long established result of Kydland and Prescott (1977) and Barro and Gordon (1983a). Their work shows that the central bank is always tempted to deviate from past policy commitments. This time-inconsistency problem precludes any *credible* policy commitment in a model with rational and perfectly informed agents.

As a side result, proposition 3 shows that time-invariant *limited* credibility is inconsistent with strategic policy deviations.

Proposition 3 *Time-invariant limited credibility ($\gamma_t = 0.5\ \forall t < T$) is inconsistent with strategic policy deviations.*

The logic of the proof is similar to the proof of proposition 1 but with $\gamma_t = 0.5\ \forall t < T$ instead of $\gamma_t = 1\ \forall t < T$.[14] $V_2^h(u_2^k) > V_2^d(u_2^k)$ requires

$$(1-a_2)^2 + \frac{\kappa^2}{\vartheta} a_2^2 < 0 \tag{29}$$

[12] Formally, if $\exists\ t$ such that $V_t^d(u_t^k) \geq V_t^h(u_t^k)$ for some k, time-invariant full credibility ($\gamma_t = 1\ \forall t < T$) is inconsistent with strategic policy deviations. The full proof is provided in the appendix, section 5.6.

[13] The full proof is provided in the appendix, section 5.6.

[14] The full proof is provided in the appendix, section 5.6.

which cannot hold with $\gamma_T = 0$. Time-invariant limited credibility is hence inconsistent with strategic policy deviations if each agent holds a correct belief about the central bank's credibility in period T. This result suggests that the time-invariant limited commitment case assumed in Debortoli et al. (2014), Debortoli and Lakdawala (2014), and Schaumburg and Tambalotti (2007) is not fully relevant under strategic policy deviations.

Are there any circumstances under which the central bank is *willing* to honor past policy commitments? The answer is yes. Proposition 4 proves that there is a threshold $\bar{\gamma}$ for which $\gamma_T \geq \bar{\gamma} \in [0, 1]$ induces the central bank to honor past policy commitments in $t \leq T$.

Proposition 4 *There exists a $\bar{\gamma} \in [0, 1]$ such that time-invariant full credibility is consistent with strategic policy deviations if $\gamma_T \geq \bar{\gamma}$.*

Proposition 4 is true if the value of honoring past policy commitments is strictly greater than the value of a policy deviation for both $t \in \{1, 2\}$ and for each potential shock sequence, given that $\gamma_T \geq \bar{\gamma} \in [0, 1]$.[15]

In period 2, the central bank is willing to honor past policy commitment if and only if $V_2^h(u_2^k) > V_2^d(u_2^k)$. With $\{u_t\}_{t=0}^T = \{H, H, H\}$, $V_2^h(u_2^k) > V_2^d(u_2^k)$ requires

$$\frac{\vartheta + \kappa^2}{\vartheta} (\Sigma + 1)^2 a_2^2 - 2\Sigma(1 + \Sigma)a_2 + \left(\Sigma^2 - \frac{\vartheta}{\vartheta + \kappa^2}\right) < 0 \tag{30}$$

where $\Sigma \equiv a_1(1 - a_0)$ is affected by a_T (and hence γ_T). In order to find $\bar{\gamma}$, guess γ_T^G and compute $\bar{a}^1$ such that equation 30 holds with equality. Then, re-arrange $\bar{a}^1$ such that

$$\bar{\gamma}^1 = \frac{(\vartheta + \kappa^2)\left(\vartheta - (\vartheta + \kappa^2)\,\bar{a}^1\right)}{\vartheta\beta\kappa^2\bar{a}^1} \tag{31}$$

The coefficients associated to a_2 in equation 30 (in particular: Σ) depend on the initial guess γ_T^G. γ_T^G is hence not necessarily equal to $\bar{\gamma}^1$ (which is found to satisfy equation 30 with equality if Σ is formed with γ_T^G). Thus we have to solve for the fixed point of $\bar{\gamma}$ in equation 30 by continued iterations.[16]

[15] Formally, if $\exists\ \gamma_T \geq \bar{\gamma} \in [0, 1]$ such that $V_1^h(\tilde{u}) > V_1^d(\tilde{u})$ and $V_2^h(\tilde{u}) > V_2^d(\tilde{u})$ for each potential shock sequence $\tilde{u}$, $\gamma_t = 1\ \forall t < T$ is consistent with strategic policy deviations. The full proof is provided in the appendix, section 5.6.

[16] Proceed as follows: First, compute the difference between γ_T^G and $\bar{\gamma}^I$ (where $\bar{\gamma}^I$ denotes $\bar{\gamma}$ after I iterations). Second, if the difference between γ_T^G and $\bar{\gamma}^I$ is above some critical value,

For reasonable parameterization, there is a $\gamma_T \geq \bar{\gamma} \in [0,1]$ such that $V_2^h(\tilde{u}) > V_2^d(\tilde{u})$ for each potential shock sequence $\tilde{u}$. Full central bank credibility is hence compatible with strategic policy deviations if a sufficient fraction of agents (inconsistently) believes that the central bank implements past policy commitments in period $T+1$.

Somewhat less rigorously, if there is uncertainty about the exact end date of the announced policy path, the central bank may find it beneficial to implement policy commitment in $t \leq T$. Though the lens of my model, this explains (at least in parts) why monetary policy commitments were often implemented in reality.

3.3 Sufficient Central Bank Credibility

What central bank credibility ($\bar{\gamma}$) sustains honored commitments in $t \leq T$? Understanding the determinants of $\bar{\gamma}$ requires the understanding of two mechanisms: First, the higher γ_T, the higher the relative cost of a policy deviation in period T (the reason is that the central bank benefits from a less severe policy trade-off, the higher its credibility). Second, the central bank is more willing to deviate, the greater the (potentially: counterfactual) inflation gap that is supposed to be implemented if past policy commitments are honored.

Taken together, these two forces suggest that a higher inflation gap under honored commitments (which increases the temptation to deviate) requires a higher $\bar{\gamma}$ (which decreases the temptation to deviate).

Figure 4 presents the difference $V_2^d(\tilde{u}) - V_2^h(\tilde{u})$, conditional on γ_2 and the realized shock sequence $\tilde{u}$. A positive difference means that the central bank prefers to deviate from past policy commitments. More specifically, blueish colors indicate values of γ_2 which support $\gamma_0 = \gamma_1 = 1$. In contrast, yellowish colors indicates values of γ_2 which are too low to incentivize the implementation of past policy commitments. The black solid line is the threshold $\bar{\gamma}$ which ensures honored commitments in $t \leq T$.

To illustrate, for the shock sequence $\{H, H, H\}$, $\gamma_2 \geq 0.49$ suffices to ensure the implementation of past policy commitments in $t \leq T$. In contrast, for the shock sequence $\{H, H, L\}$, $\gamma_2 \geq 0.72$ is necessary to avoid $\gamma_1 = 0$. The threshold for $\bar{\gamma}$ hence carries information about the central bank's temptation to deviate from the announced policy path.

Why is it that $\bar{\gamma}$ has to be higher for $\{H, H, L\}$ than for $\{H, H, H\}$? The reason is that the central bank commits to a conditional future deflation (in-

repeat the computation of $\bar{a}^I$ (and the corresponding $\bar{\gamma}^I$) with γ^{I-1} as an input. Repeat until γ^I is sufficiently close to $\bar{\gamma}^I$ and report $\bar{\gamma} = \bar{\gamma}^I$.

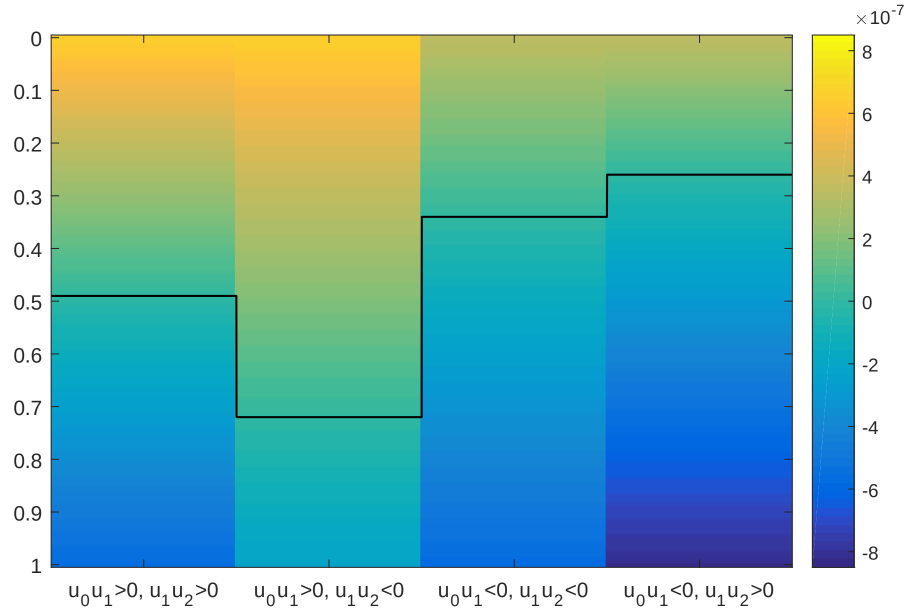

Figure 4: Heatmap: Threshold γ_2 to have $\gamma_0 = \gamma_1 = 1$

flation) in the presence of a (sequence of) positive (negative) cost-push shocks. The conditional future deflation is amplified if the cost-push shock changes its sign between the current and the future period. More specifically, after two consecutive positive cost-push shocks, the expected inflation rate for period 2 under honored commitments is negative. A negative cost-push shock in period 2 amplifies the (conditional) commitment to a deflation, i.e. the change in the sign of the cost-push shock between period 1 and 2 makes the inflation gap under honored commitments greater (compared to a shock sequence in which three consecutive positive cost-push shocks materialize). The central bank is thus more inclined to deviate from past policy commitments if the shock sequence $\{H, H, L\}$ materializes. Because the central bank is more tempted to deviate under $\{H, H, L\}$ than under $\{H, H, H\}$ (at some constant γ_2), $\bar{\gamma}$ must be higher under $\{H, H, L\}$.

Figure 5 displays the welfare under honored commitments for the shock sequence $\{H, H, L\}$ and the shock sequence $\{H, H, H\}$. As discussed above, the central bank prefers $\{H, H, H\}$ over $\{H, H, L\}$ if it honors past policy commitments. This suggests that the central bank faces a state dependent *temptation* to deviate from past policy commitments.[17] Put differently, my model suggests that the central bank may face more or less (political) pressure to renege on past promises depending on the state of the economy.

[17]This does not contradict proposition 3 which says that a time-invariant limited credibility is inconsistent with strategic policy deviations. The reason is that past policy commitments are *never* (*always*) honored if $\gamma_T < \bar{\gamma}$ ($\gamma_T > \bar{\gamma}$).

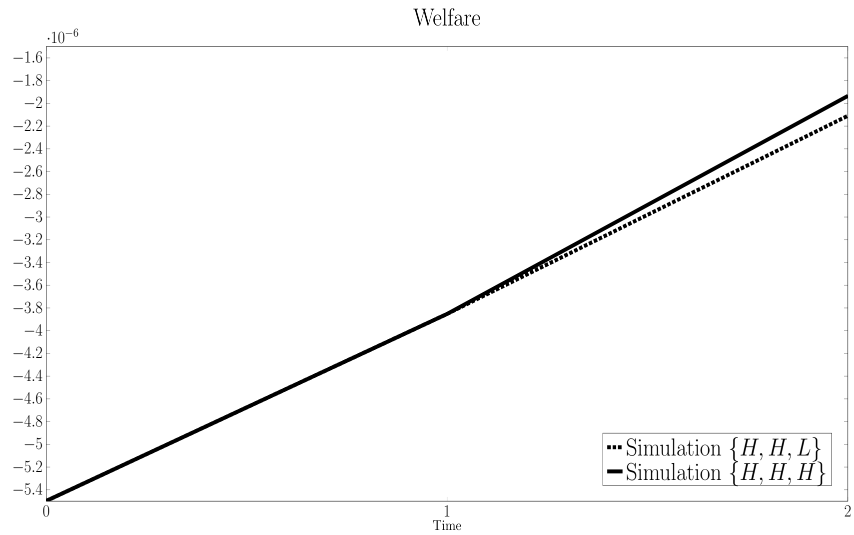

Figure 5: Welfare Under Honored Commitments

Of course, for $\gamma_0 = \gamma_1 = 1$ to be consistent with rational expectations, the central bank must already decide to implement past policy commitments in period 1 with probability 1. Moreover, γ_2 must be such that the central bank is known to be willing to implement policy commitments in period 2 with certainty, i.e. even if the realized u_2 is such that the temptation to renege in period 2 is maximized. The central bank is willing to honor past policy commitments under any realization of the shock sequence if $\gamma_2 \geq 0.72$.[18]

4 Concluding Remarks

The empirical motivation for this paper is the observation that central banks have recently used more or less explicit policy commitments to manage public expectations. Woodford (2012) and Andrade et al. (2016) argue that these commitments are subject to potential revisions. Current and future credibility therefore plays a crucial role in determining the effectiveness of monetary policy commitments. In particular, if the central bank's credibility is high, agents are more willing to adjust their expectations. The responsiveness

[18]Because the agents know the first two realizations of the shock sequence in $t = 1$, the consistent γ_1 may depend on the state of the economy. In particular, after two consecutive, equally signed cost-push shocks, $\gamma_1 = 0.5$ is consistent with honored commitments in period 1 in some range of γ_T, while $\gamma_1 = 1$ is consistent in the *same* range of γ_T if the sign of the cost-push shock differs between period 0 and 1 (this is true for $\gamma_T \in (0.49, 0.72)$). Central bank credibility in period 1 may be 0.5 because the inflation rate in period 2 under honored commitments, and with it the temptation to deviate, differs with the first two realizations of the cost-push shock. I constrain the attention to cases in which $\gamma_2 \geq \bar{\gamma}$ implies $\gamma_1 = 1$ for each potential shock sequence to avoid dealing with path dependent sequences of central bank credibilities. $\bar{\gamma} = 0.72$ is hence the most conservative measure of the threshold $\bar{\gamma}$ that is necessary to defer policy deviations in $t \leq T$.

of the agents' expectations feeds back into the optimal behavior of the central bank, i.e. affects the optimal monetary policy rule.

Most papers in the recent literature on limited commitment in monetary policy, e.g. Debortoli et al. (2014) and Bodenstein et al. (2012), make two critical assumption: First, central bank credibility is time-invariant and exogenous. Second, policy deviations occur randomly. My paper allows for time-variation in the central bank's credibility and endogenizes the monetary policy decision. It asks two questions: First, how does future central bank credibility affect the optimal monetary policy rule? Second, under which circumstances is it optimal for the central bank to implement past policy commitments?

To address this question, I use a simple New Keynesian dynamic stochastic general equilibrium (DSGE) model in which the central bank decides strategically if it wants to honor past policy commitments. Policy deviation come with a permanent and complete loss of central bank credibility, i.e. higher macroeconomic volatility in the future. The central bank decides to renege on past policy commitments if and only if a deviation delivers a higher welfare than the implementation of past policy commitments.

My results show that higher future expected central bank credibility attenuates the current period policy trade-off between a stable inflation rate and a stable output gap. Moreover, it decreases the optimal persistence in the output gap. Both is because agents are more willing to adjust their expectations if policy commitment are expected to be honored with a higher probability and/or over a longer horizon. The higher the future expected central bank credibility, the lower the cost of implementing policy commitments today.

The less costly implementation of the optimal monetary policy rule under high credibility, together with the loss of credibility after a policy deviation, provides the central bank with a incentive to implement past policy commitments. My main result shows that the central bank is willing to implement past policy commitments if a sufficient fraction of agents is not aware of the exact end date of the policy commitment. This finding challenges the time-inconsistency argument against monetary policy commitments and provides a potential explanation for the repeated implementation of monetary policy commitments in reality.

Further research is desirable along multiple dimensions of the model. First, the rather conservative assumption about the complete and permanent loss of credibility after a policy deviation could be relaxed. In particular, the central bank should, in some way, be enabled to restore its credibility after a policy deviation. Second, the model could be generalized to study path dependent evolutions of central bank credibilities. Third, one could introduce evolutionary stable preferences to analyze whether a central bank would abstain from policy deviations under complete information.

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5 Appendix

The appendix provides details on the derivation of the New-Keynesian dynamic stochastic general equilibrium (DSGE) model. Also, it contains the full proofs the propositions presented in the main text. The non-policy equilibrium is based on Galí (2015) and Canetg (2015), the second order approximation to the household's utility function on Galí (2015) and Woodford (2005), the derivation of the central bank objective function under limited commitment on Debortoli and Nunes (2010), and the model solution under limited commitment on Debortoli et al. (2014) and Galí (2015).

5.1 Non-Policy Equilibrium

This section derives the non-policy equilibrium of the simple New Keynesian model.

Households: The (representative) household maximizes its expected discounted utility subject to a sequence of dynamic budget constraints and a No-Ponzi condition.

The intertemporal utility function is additively separable in a consumption bundle (C_t) and leisure ($1 - N_t$). $\beta \in (0, 1)$ is the discount factor of the household, $\sigma \geq 0$ the constant relative risk aversion or, equivalently, the inverse intertemporal elasticity of substitution, and $\varphi \geq 0$ the inverse of the Frisch labor supply elasticity, i.e. the inverse wage elasticity of hours worked (N_t). The (sequence of) dynamic budget constraints are associated to the Lagrange multiplier(s) η_t. P_t denotes the price level, Q_t the period t price of a risk-free security B_{t+1} which pays one unit in period $t + 1$, W_t the nominal wage, and D_t the (aggregated) nominal firm profit. The Lagrangian is given by

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) - \eta_t (P_t C_t + Q_t B_{t+1} - W_t N_t - D_t - B_t) \right] \tag{32}$$

The No-Ponzi condition prevents excessive debt, i.e. ensures that the household can repay its debt in every period.[19] $\Lambda_{T,t} \equiv \beta^{T-t} \frac{U_{C,T}}{U_{C,t}}$ is the stochastic discount factor, cf. Galí (2015, 56, 84).

$$\lim_{T \to \infty} \mathbb{E}_0 \left[\Lambda_{T,0} \frac{B_T}{P_T} \right] \geq 0 \tag{33}$$

[19] In equilibrium, bond holdings B_t are zero because all households are equal.

The household takes prices (P_t, Q_t, W_t) and nominal firm profits (D_t) as given when solving its maximization problem. The first order conditions with respect to C_t, N_t, and B_{t+1} are

$$\beta^t C_t^{-\sigma} = \beta^t \eta_t \tag{34}$$
$$\beta^t N_t^{\varphi} = \beta^t \eta_t \tag{35}$$
$$\beta^t \eta_t = \beta^{t+1} \mathbb{E}_t \eta_{t+1} \tag{36}$$

The transversality condition is

$$\lim_{T\to\infty} \mathbb{E}_0 \left[\Lambda_{T,0} \frac{B_T}{P_T} \right] = 0 \tag{37}$$

combining the first order conditions yields

$$N_t^{\varphi} C_t^{\sigma} = \frac{W_t}{P_t} \tag{38}$$
$$Q_t = \beta \mathbb{E}_t \left(\frac{P_t}{P_{t+1}} \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \right) \tag{39}$$

Equation 38 constitutes a household labor market optimum [LM], i.e. an optimal labor supply. It shows that hours worked react positively to the real wage W_t/P_t and more strongly to a change in the (real and nominal) wage, the lower φ (the lower the inverse wage elasticity of hours worked).

Equation 39 is the Euler equation [EE]. It postulates equality between the marginal utility of consuming an additional Q_t units today and the discounted marginal utility of consuming one additional unit tomorrow. The intuition is as follows. The household can either consume or save an additional Q_t units today. If the household consumes an additional Q_t units today, it gets $Q_t C_t^{-\sigma}/P_t$ utils from it. If the household saves an additional Q_t units today, it has one additional unit to consume tomorrow. Because the household is impatient, getting $C_{t+1}^{-\sigma}/P_{t+1}$ utils tomorrow is only worth $\beta C_{t+1}^{-\sigma}/P_{t+1}$ utils today. If the marginal utilities of the two options (consuming an additional Q_t units today and saving an additional Q_t units today) were not the same, the household could increase its utility by re-allocating consumption (e.g. consume more today and less tomorrow). An optimal intertemporal allocation of consumption thus requires the marginal utilities of the two options to be equal.

Bundler: Suppose that there is an intermediary firm (the bundler) which bundles the intermediary consumption goods C_{it} into a single good C_t (output). I assume $\epsilon \in (1, \infty)$, i.e. some, but imperfect substitutability among intermediary consumption goods ("a taste for variety").[20] More specifically, I impose the Dixit-Stiglitz (Dixit and Stiglitz (1977)) aggregator, assuming a continuum of goods on the interval $[0, 1]$.

$$C_t = \left(\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{40}$$

The (static) nominal profit maximization problem of the bundler is

$$\max_{C_{it}} \left\{ P_t \left(\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 P_{it} C_{it} di \right\} \tag{41}$$

The optimal demand for the bundler's input good C_{it} is

$$P_t C_t^{\frac{1}{\epsilon}} C_{it}^{-\frac{1}{\epsilon}} - P_{it} = 0 \tag{42}$$

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\epsilon} C_t \tag{43}$$

i.e. a decreasing function of its price P_{ti} and an increasing function of P_t and C_t.

Firms: There is a continuum of monopolistically competitive firms on the interval $[0, 1]$, each producing a single differentiated intermediary output good Y_{it} with a linear technology $Y_{it} = N_{it}$ (with N_{it} being the labor demand of firm i). As in Calvo (1983), in every period a firm decides on a new price P_{ti} of its output good with probability $1 - \theta$. If selected to change its price, firm i maximizes the period t market value of current and discounted future expected real profits over P_t^*.[21] Future profits are weighted by the probability of seeing P_t^* in effect in that period (θ^i). As in Galí (2015, 84), the optimal price setting problem takes the following form.

[20] ϵ is the elasticity of substitution among intermediary goods. More specifically, intermediary goods are perfect complements if $\epsilon \to 0$ and perfect substitutes if $\epsilon \to \infty$. $\epsilon \in (0, 1)$ implies that an increased variety *reduces* utility ("distaste for variety"), cf. Tutorial Dixit Stigliz I (Franck Portier's lecture notes). With $\epsilon = 1$, preferences are of the Cobb-Douglas form, cf. Tutorial Dixit Stigliz II. Preferences are convex if $\epsilon \in (0, \infty)$.

[21] cf. Galí (2015, 56).

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right] \tag{44}$$

$Y_{t+k|t}$ is the output of a firm that last reset its price in period t and $\mathcal{C}_{t+k}(Y_{t+k|t})$ is the associated nominal cost function. Use the constant elasticity of substitution $\epsilon \equiv -\frac{dY_{t+k|t}}{dP_t^*}\frac{P_t^*}{Y_{t+k|t}}$, $\Psi_{t+k|t} \equiv \frac{d\mathcal{C}_{t+k}(Y_{t+k|t})}{dY_{t+k|t}}$ (the nominal marginal costs of a firm that last reset its price in period t), and the definition of the steady state mark-up $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$ to write the firm's first order condition as

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(Y_{t+k|t} + P_t^* \frac{dY_{t+k|t}}{dP_t^*} - \frac{d\mathcal{C}_{t+k}(Y_{t+k|t})}{dY_{t+k|t}} \frac{dY_{t+k|t}}{dP_t^*} \right) \right] = 0 \tag{45}$$

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(1 + \frac{dY_{t+k|t}}{dP_t^*} \frac{P_t^*}{Y_{t+k|t}} - \Psi_{t+k|t} \frac{dY_{t+k|t}}{dP_t^*} \frac{1}{Y_{t+k|t}} \right) Y_{t+k|t} \right] = 0 \tag{46}$$

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(1 - \epsilon + \Psi_{t+k|t} \epsilon \frac{1}{P_t^*} \right) Y_{t+k|t} \right] = 0 \tag{47}$$

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(P_t^* - \mathcal{M} \Psi_{t+k|t} \right) Y_{t+k|t} \right] = 0 \tag{48}$$

By the definition of the log-expressions for the variables, we have that $exp(p_t^* - p_{t+k}) \equiv \frac{P_t^*}{P_{t+k}}$ and $exp(\psi_{t+k|t} - p_{t+k}) \equiv \frac{\Psi_{t+k|t}}{P_{t+k}}$. Furthermore, $log(\mathcal{M}) \equiv \mu^{MC}$ such that

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\Lambda_{t,t+k} \left(exp(p_t^* - p_{t+k}) - exp(\psi_{t+k|t} - p_{t+i} + \mu^{MC}) \right) Y_{t+k|t} \right] = 0 \tag{49}$$

A first order Taylor expansion around a zero inflation steady state (characterized by $p_t^* - p_{t+k} = 0$, $\Lambda_{t,t+k} = \beta^i$, $Y_{t+k|t} = Y$ and, consequently, $p_{t+k} = \psi_{t+k|t} + \mu^{MC}$, cf. table 5) yields

$$\sum_{k=0}^{\infty} (\theta\beta)^k Y (exp(0) - exp(0)) \mathbb{E}_t \left[(p_t^* - p_{t+k} - 0) - (\psi_{t+k|t} - p_{t+k} + \mu^{MC} - 0)) \right] = 0 \tag{50}$$

$$\sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t \left[p_t^* - \psi_{t+k|t} - \mu^{MC} \right] = 0 \tag{51}$$

$$\frac{1}{1-\theta\beta}(p_t^* - \mu^{MC}) = \sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t \psi_{t+k|t} \tag{52}$$

$$p_t^* = \mu^{MC} + (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t \psi_{t+k|t} \tag{53}$$

Because the marginal product of labor is one, we have $\Psi_{t+k|t} = W_{t+k}$, i.e. $\Psi_{t+k|t}$ is independent of when the price of the intermediary good was last changed. Hence, we can write $\psi_{t+k|t} = \psi_{t+k} \equiv p_{t+k} - \mu^{MC}_{t+k}$. The above expression for p_t^* can be written recursively as

$$p_t^* = \mu^{MC} + (1-\theta\beta)\psi_t + (1-\theta\beta)\theta\beta \sum_{k=0}^{\infty} \mathbb{E}_t(\theta\beta)^k \psi_{t+1+k} \tag{54}$$

$$p_t^* = \mu^{MC} + (1-\theta\beta)(p_t - \mu_t) + \theta\beta \left[\mathbb{E}_t p_{t+1}^* - \mu^{MC}\right] \tag{55}$$

$$p_t^* = (1-\theta\beta)p_t - (1-\theta\beta)\hat{\mu}_t^{MC} + \theta\beta\mathbb{E}_t p_{t+1}^* \tag{56}$$

$$(1-\beta\theta L^{-1})p_t^* = (1-\theta\beta)p_t - (1-\theta\beta)\hat{\mu}_t^{MC} \tag{57}$$

with $\hat{\mu}_t^{MC} = \mu_t^{MC} - \mu^{MC}$. From the definition of the aggregated price level (cf. Galí (2015, 55)), it follows that

$$p_t = \theta p_{t-1} + (1-\theta)p_t^* \tag{58}$$

$$p_t^* = \frac{1}{1-\theta}p_t - \frac{\theta}{1-\theta}p_{t-1} \tag{59}$$

Combining the two equations yields

$$(1-\beta\theta L^{-1})\left(\frac{1}{1-\theta}p_t - \frac{\theta}{1-\theta}p_{t-1}\right) = (1-\theta\beta)(p_t - \hat{\mu}_t^{MC}) \tag{60}$$

$$p_t - \beta\theta\mathbb{E}_t p_{t+1} - (\theta p_{t-1} - \beta\theta^2 p_t) = (1-\theta)(1-\theta\beta)(p_t - \hat{\mu}_t^{MC}) \tag{61}$$

$$p_t - p_{t-1} = \beta\left[\mathbb{E}_t p_{t+1} - p_t\right] - \frac{(1-\theta)(1-\theta\beta)}{\theta}\hat{\mu}_t^{MC} \tag{62}$$

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} - \xi\hat{\mu}_t^{MC} \tag{63}$$

with $\xi \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.[22]

Equilibrium: The following system of equations determines the non-policy equilibrium.[23]

[22] In general, $\xi \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$ with $\Theta \equiv \frac{1-\alpha}{1-\alpha(1-\epsilon)}$.

[23] Use $-log(\beta) \equiv \rho$ and $-log(Q_t) \equiv i_t$ when taking the log of the Euler equation (equation 39).

Equation	**Name**	**Source**
$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1} - \rho)$	[EE]	Households: Euler equation
$\varphi n_t = w_t - p_t - \sigma c_t$	[LM]	Households: Labor supply equation
$y_t = n_t$	[PF]	Firms: Production function
$\pi_t = \beta \mathbb{E}_t \pi_{t+1} - \xi \hat{\mu}_t^{MC}$	[PK]	Firms: Optimal price setting
$y_t = c_t$	[MCC]	Market clearing condition

Table 1: Non-Policy Equilibrium, in logs, Version I

which can be simplified to

Equation	**Name**	**Description**
$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1} - \rho)$	[IS]	New Keynesian IS-Curve
$\pi_t = \beta \mathbb{E}_t \pi_{t+1} - \xi \hat{\mu}_t^{MC}$	[PK]	New Keynesian Phillips Curve
$(\sigma + \varphi) y_t = w_t - p_t$	[LM,PF]	Labor market equilibrium

Table 2: Non-Policy Equilibrium, in logs, Version II

Use the definition of the firm mark-up $\mu_t^{MC} \equiv p_t - \psi_t$, $\psi_t = w_t$ (from $\Psi_{t+k|t} \equiv \frac{d\mathcal{C}_{t+k}(Y_{t+k|t})}{dY_{t+k|t}}$ and the fact that the marginal product of labor is 1) and [LM,PF] to get

$$\mu_t^{MC} \equiv p_t - \psi_t \tag{64}$$
$$\mu_t^{MC} \equiv p_t - w_t \tag{65}$$
$$\mu_t^{MC} \equiv p_t - ((\sigma + \varphi) y_t + p_t) \tag{66}$$
$$\mu_t^{MC} = -(\sigma + \varphi) y_t \tag{67}$$

Furthermore, with flexible prices, $\theta = 0$ (cf. Galí (2015, 62)) such that the firm's first order condition collapses to $\mu^{MC} = p_t^* + \psi_t$. Combined with $\psi_t = w_t$ (from $\Psi_{t+k|t} \equiv \frac{d\mathcal{C}_{t+k}(Y_{t+k|t})}{dY_{t+k|t}}$ and the fact that the marginal product of labor is 1) and using [LM,PF] yields

$$\mu^{MC} = -(\sigma + \varphi) y_t^n \tag{68}$$

i.e. a constant natural output level (which in turn implies $y_t^n = y$ in every period). Consequently

$$\hat{\mu}_t^{MC} = -(\sigma + \varphi)(y_t - y_t^n) \tag{69}$$

and

Equation	**Name**	**Description**
$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - \rho\right)$	[IS]	New Keynesian IS-Curve
$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \tilde{y}_t$	[PK]	New Keynesian Phillips Curve

Table 3: Non-Policy Equilibrium, in terms of $\tilde{y}_t$

with $\tilde{y}_t \equiv y_t - y_t^n$ and $\kappa \equiv \xi(\sigma + \frac{\varphi+\alpha}{1-\alpha})$. Acknowledge that $\mu^{MC} = 0$ defines the efficient output level such that $\mathbb{E}_t y_{t+1}^e = y_t^e = 0$. Define $x_t \equiv y_t - y_t^e$ and $u_t \equiv \kappa(y_t^e - y_t^n)$ (as in Galí (2015, 128)), and the non-policy equilibrium follows.[24]

Equation	**Name**	**Description**
$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - \rho\right)$	[IS]	New Keynesian IS-Curve
$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t$	[PK]	New Keynesian Phillips Curve

Table 4: Non-Policy Equilibrium, in terms of x_t

5.2 Steady State

This section derives the steady state of the New Keynesian model.

No.	**Formally**	**Reason**
1)	$C_t = C_{t+i}$	No driving force is growing over time
2)	$P_t = P_{t+i}$	By the definition of a zero inflation steady state
3)	$\Lambda_{t,t+i} = \beta^i$	Follows from 1) and 2)
4)	$P_t^* = P_{t-1}$	Follows from 2) and the definition of the aggregated price level
5)	$Y_{t+i\|t} = Y$	Follows from 1) and 4), no price dispersion in steady state
6)	$\Psi_{t+i\|t} = \Psi_t$	Follows from 1) and 4), no price dispersion in steady state
7)	$P_t = \mathcal{M}\Psi_t$	Follows from 3), 4), 5), 6) and the firm's first order condition

[24] The cost-push shock u_t must thus be related to changes in μ^{MC}, σ, φ, or κ.

Table 5: Characteristics of the Zero Inflation Steady State

The steady state log-output level is determined by the household's labor market optimality condition, the market clearing condition, the production function, and the steady state log real wage

$$\varphi n_t = w_t - p_t - \sigma c_t \tag{70}$$

$$\varphi y_t = w_t - p_t - \sigma y_t \tag{71}$$

$$(\sigma + \varphi) y_t = w_t - p_t \tag{72}$$

$$y = -\frac{\mu^{MC}}{\sigma + \varphi} \tag{73}$$

The steady state log-consumption level is determined by the market clearing condition and the steady state log-output level

$$c = y \tag{74}$$

$$c = -\frac{\mu^{MC}}{(\sigma + \varphi)} \tag{75}$$

The steady state log hours worked is determined by the production function and the steady state log-output level

$$n = y \tag{76}$$

$$n = -\frac{\mu^{MC}}{(\sigma + \varphi)} \tag{77}$$

The steady state inflation rate is determined by the definition of the zero inflation steady state

$$\pi = 0 \tag{78}$$

The steady state nominal interest rate is determined by the Euler equation [EE], the fact that in steady state $C_t = C_{t+i}$, and the steady state inflation rate. Use $-log(Q_t) \equiv i_t$ and $-log(\beta) \equiv \rho$.

$$log(Q_t) = log(\beta) \tag{79}$$

$$i = \rho \tag{80}$$

The steady state real interest rate is determined by the Fisher equation and the steady state inflation rate

$$r_t \equiv i_t - \mathbb{E}_t \pi_{t+1} \tag{81}$$

$$r = \rho \tag{82}$$

5.3 Approximating the Household's Utility

This subsection presents the derivation of a second order approximation to the household's utility function. Using the approximated household's utility function simplifies the monetary policy problem because it expresses the welfare loss in terms of the welfare relevant output gap and the inflation rate. The derivation follows Galí (2015, 85, 117, 154) and Woodford (2005, 399, 694). A second order approximation to the household's felicity function ("current period utility function") around the steady state yields

$$\begin{aligned} U_t - U =& U_C C \left(\frac{C_t - C}{C} \right) + U_N N \left(\frac{N_t - N}{N} \right) \\ &+ \frac{1}{2} U_{CC} C^2 \left(\frac{C_t - C}{C} \right)^2 + \frac{1}{2} U_{NN} N^2 \left(\frac{N_t - N}{N} \right)^2 \end{aligned} \tag{83}$$

Up to a second order approximation $\frac{X_t - X}{X} = \hat{x}_t + \frac{1}{2}\hat{x}_t^2$, with $\hat{x}_t \equiv x_t - x \equiv log\left(\frac{X_t}{X}\right)$. Further use $\sigma \equiv -\frac{U_{CC}}{U_C} C$ (the inverse intertemporal elasticity of substitution) and $\varphi \equiv \frac{U_{NN}}{U_N} N$ (the inverse of the Frisch labor supply elasticity) to write

$$U_t - U = U_C C \left(\hat{y}_t + \frac{1-\sigma}{2} \hat{y}_t^2 \right) + U_N N \left(\hat{n}_t + \frac{1+\varphi}{2} \hat{n}_t^2 \right) \tag{84}$$

Next, realize that aggregated employment is the sum of employment across firms $N_t = \int_0^1 N_{ti} di$. Combine this with $C_{ti} = Y_{ti}$ (the market clearing condition which holds on the firm level and in the aggregate), $C_{ti} = \left(\frac{P_{ti}}{P_t}\right)^{-\epsilon} C_t$, and the (strictly concave and total factor productivity augmented) production function.[25]

$$Y_{ti} = A_t N_{ti}^{1-\alpha} \tag{85}$$
$$N_{ti} = \left(\frac{Y_{ti}}{A_t}\right)^{\frac{1}{1-\alpha}} \tag{86}$$
$$N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_{ti}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di \tag{87}$$

Take logs and subtract the (log) production function evaluated at the steady state.

$$\hat{n}_t = \frac{1}{1-\alpha}(\hat{y}_t - a_t - d_t) \tag{88}$$

with $d_t \equiv (1-\alpha) log \int_0^1 \left(\frac{P_{ti}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di.$

Approximating d_t consists of three steps. First, from Galí (2015, 53), $\int_0^1 P_{it}C_{it}di = P_tC_t$ implies an aggregated price level of

$$P_t \equiv \left(\int_0^1 P_{ti}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}} \tag{89}$$
$$1 = \int_0^1 exp((1-\epsilon)(p_{ti} - p_t))di \tag{90}$$

A second order approximation to the aggregated price level equation yields

$$1 = 1 + 1(1-\epsilon)\hat{p}_{ti} + \frac{(1-\epsilon)^2}{2}\hat{p}_{ti}^2 \tag{91}$$
$$\mathbb{E}_i\left(\hat{p}_{ti}\right) = \frac{(\epsilon-1)}{2}\mathbb{E}_i\left(\hat{p}_{ti}^2\right) \tag{92}$$

with $\hat{p}_{ti} \equiv p_{ti} - p_t$. Second, the expression $\int_0^1 \left(\frac{P_{ti}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di$ equals

[25] Using $Y_{ti} = A_t N_{ti}^{1-\alpha}$ instead of $Y_{ti} = N_{ti}$ is without loss of generality since the later is a special case of the former (with $A_t = 1$ and $\alpha = 0$).

$$\int_0^1 \left(\frac{P_{ti}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di = \mathbb{E}_i \left[\left(\frac{P_{ti}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} \right] \tag{93}$$

$$\int_0^1 \left(\frac{P_{ti}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di = \mathbb{E}_i \left[exp\left(-\frac{\epsilon}{1-\alpha}\hat{p}_{ti}\right) \right] \tag{94}$$

which is, up to a second order approximation and using the insight from before (equation 92) equal to

$$\mathbb{E}_i \left[exp\left(-\frac{\epsilon}{1-\alpha}\hat{p}_{ti}\right) \right] = \mathbb{E}_i \left[1 - \frac{\epsilon}{1-\alpha}\hat{p}_{ti} + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2 \hat{p}_{ti}^2 \right] \tag{95}$$

$$\mathbb{E}_i \left[exp\left(-\frac{\epsilon}{1-\alpha}\hat{p}_{ti}\right) \right] = 1 - \frac{\epsilon}{1-\alpha}\mathbb{E}_i\left(\hat{p}_{ti}\right) + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2 \mathbb{E}_i\left(\hat{p}_{ti}^2\right) \tag{96}$$

$$\mathbb{E}_i \left[exp\left(-\frac{\epsilon}{1-\alpha}\hat{p}_{ti}\right) \right] = 1 + \frac{1}{2}\frac{\epsilon}{1-\alpha}\frac{1}{\Theta}\mathbb{E}_i\left(\hat{p}_{ti}^2\right) \tag{97}$$

with $\Theta \equiv \frac{1-\alpha}{1-\alpha(1-\epsilon)}$. Third, $\mathbb{E}_i\left(\hat{p}_{ti}^2\right)$ equals, up to a second order approximation, using $x = p_{ti} - p_t$ (with $f''(x) = 0$) and $x_0 = p_{ti} - \mathbb{E}_i(p_{ti})$,

$$\mathbb{E}_i\left(\hat{p}_{ti}^2\right) = \mathbb{E}_i\left[\left(p_{ti} - \mathbb{E}_i\left(p_{ti}\right)\right)^2 + 2\left(p_{ti} - \mathbb{E}_i\left(p_{ti}\right)\right)\left(p_t - \mathbb{E}_i p_{ti}\right) \right] \tag{98}$$

$$\mathbb{E}_i\left(\hat{p}_{ti}^2\right) = \mathbb{E}_i\left[\left(p_{ti} - \mathbb{E}_i\left(p_{ti}\right)\right)^2 \right] \tag{99}$$

$$\mathbb{E}_i\left(\hat{p}_{ti}^2\right) = V_i\left(p_{ti}\right) \tag{100}$$

Plugging this into the insight from before (equation 97) and taking the log yields

$$log\left\{ \mathbb{E}_i \left[exp\left(-\frac{\epsilon}{1-\alpha}\hat{p}_{ti}\right) \right] \right\} = log\left\{ 1 + \frac{1}{2}\frac{\epsilon}{1-\alpha}\frac{1}{\Theta}V_i\left(p_{ti}\right) \right\} \tag{101}$$

$$d_t = (1-\alpha) log\left\{ 1 + \frac{1}{2}\frac{\epsilon}{1-\alpha}\frac{1}{\Theta}V_i\left(p_{ti}\right) \right\} \tag{102}$$

which equals, up to a second order approximation, using $x = \frac{1}{2}\frac{\epsilon}{1-\alpha}\frac{1}{\Theta}V_i\left(p_{ti}\right)$ and $x_0 = 0$, and realizing that $V_i(p_{ti})^2$ is higher than second order,

$$d_t = (1-\alpha)\left\{ \frac{1}{2}\frac{\epsilon}{1-\alpha}\frac{1}{\Theta}V_i\left(p_{ti}\right) \right\} \tag{103}$$

$$d_t = \frac{1}{2}\frac{\epsilon}{\Theta}V_i\left(p_{ti}\right) \tag{104}$$

Back to the approximated felicity function: Combine equation 84 and 88 with the approximation for d_t (equation 104), ignoring terms of third and higher order, and use that in an efficient steady state $-\frac{U_N}{U_C} = MPN$ with $MPN = (1-\alpha)\frac{Y}{N}$ (cf. Galí (2015, 21, 23)).

$$U_t - U = U_C C\left(\hat{y}_t + \frac{1-\sigma}{2}\hat{y}_t^2\right) + \frac{U_N N}{1-\alpha}\left(\hat{y}_t + d_t + \frac{1+\varphi}{2(1-\alpha)}(\hat{y}_t - a_t)^2\right) \tag{105}$$

$$\frac{U_t - U}{U_C C} = -\frac{1}{2}\left[\frac{\epsilon}{\Theta}V_i\left(p_{ti}\right)\underbrace{-(1-\sigma)\hat{y}_t^2 + \frac{1+\varphi}{1-\alpha}(\hat{y}_t - a_t)^2}_{z}\right] \tag{106}$$

Using $\hat{y}_t^n = \hat{y}_t^e = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}a_t$ (cf. Galí (2015, 62, 155) and equation 68) we can express z as

$$z = \frac{1+\varphi}{1-\alpha}(\hat{y}_t - a_t)^2 - (1-\sigma)\hat{y}_t^2 \tag{107}$$

$$z = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\hat{y}_t^2 - 2\frac{1+\varphi}{1-\alpha}\hat{y}_t a_t \tag{108}$$

$$z = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(\hat{y}_t^2 - 2\hat{y}_t\hat{y}_t^e) \tag{109}$$

$$z = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(\hat{y}_t - \hat{y}_t^e)^2 \tag{110}$$

Recall that $x_t \equiv y_t - y_t^e$ (with $x = y - y^e$) such that $\hat{x}_t = \hat{y}_t - \hat{y}_t^e$.

$$z = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)x_t^2 \tag{111}$$

Using the above result in equation 106 yields

$$\frac{U_t - U}{U_C C} = -\frac{1}{2}\left[\frac{\epsilon}{\Theta}V_i\left(p_{ti}\right) + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)x_t^2\right] \tag{112}$$

Let us now turn to $V_i\left(p_{ti}\right)$. Claim: Ignoring terms of third and higher order, $V_i\left(p_{ti}\right) = \theta V_i\left(p_{t-1,i}\right) + \frac{\theta}{1-\theta}\pi_t^2$. Proof, following Woodford (2005, 399, 694): Let $\bar{P}_t \equiv \mathbb{E}_i(p_{ti})$

$$\bar{P}_t - \bar{P}_{t-1} = \mathbb{E}_i\left(p_{ti} - \bar{P}_{t-1}\right) \tag{113}$$
$$\bar{P}_t - \bar{P}_{t-1} = \theta\mathbb{E}_i\left(p_{t-1,i} - \bar{P}_{t-1}\right) + (1-\theta)\left(p_t^* - \bar{P}_{t-1}\right) \tag{114}$$
$$\bar{P}_t - \bar{P}_{t-1} = (1-\theta)\left(p_t^* - \bar{P}_{t-1}\right) \tag{115}$$

Similarly, using the above result

$$V_i\left(p_{ti}\right) = V_i\left(p_{ti} - \bar{P}_{t-1}\right) \tag{116}$$
$$V_i\left(p_{ti}\right) = \mathbb{E}_i\left[\left(p_{ti} - \bar{P}_{t-1}\right)^2\right] - \left[\mathbb{E}_i\left(p_{t-1,i} - \bar{P}_{t-1}\right)\right]^2 \tag{117}$$
$$V_i\left(p_{ti}\right) = \theta\mathbb{E}_i\left[\left(p_{t-1,i} - \bar{P}_{t-1}\right)^2\right] + (1-\theta)(p_t^* - \bar{P}_{t-1})^2 - \left(\bar{P}_t - \bar{P}_{t-1}\right)^2 \tag{118}$$
$$V_i\left(p_{ti}\right) = \theta V_i\left(p_{t-1,i}\right) - \frac{(\bar{P}_t - \bar{P}_{t-1})^2}{1-\theta} - \left(\bar{P}_t - \bar{P}_{t-1}\right)^2 \tag{119}$$
$$V_i\left(p_{ti}\right) = \theta V_i\left(p_{t-1,i}\right) + \frac{\theta}{1-\theta}(\bar{P}_t - \bar{P}_{t-1})^2 \tag{120}$$
$$V_i\left(p_{ti}\right) = \theta V_i\left(p_{t-1,i}\right) + \frac{\theta}{1-\theta}\pi_t^2 \tag{121}$$

q.e.d. Use $V_i\left(p_{ti}\right) \equiv \Delta_t$ and iterate the above equation backwards.

$$\Delta_t = \theta\Delta_{t-1} + \frac{\theta}{1-\theta}\pi_t^2 \tag{122}$$
$$\Delta_t = \theta^{t+1}\Delta_{-1} + \left(\frac{\theta}{1-\theta}\right)\sum_{s=0}^{t}\theta^{t-s}\pi_s^2 \tag{123}$$
$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \left(\frac{\theta}{1-\theta}\right)\sum_{t=0}^{\infty}\beta^t\sum_{s=0}^{t}\theta^{t-s}\pi_s^2 \tag{124}$$
$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{\theta}{1-\theta}\left[\pi_0^2 + \beta\theta\left(\pi_0^2 + \frac{\pi_1^2}{\theta}\right) + (\beta\theta)^2\left(\pi_0^2 + \frac{\pi_1^2}{\theta} + \frac{\pi_2^2}{\theta^2}\right) + \ldots\right] \tag{125}$$
$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{\theta}{(1-\theta)(1-\beta\theta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 \tag{126}$$

Back to the approximated felicity function, i.e. equation 112. Multiplied by (-1) and the expectation operator $\mathbb{E}_0$, summed over each (current and future) period, and appropriately discounted by β^t

$$-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\frac{U_t - U}{U_C C} = \frac{1}{2}\mathbb{E}_0\left[\frac{\epsilon}{\Theta}\sum_{t=0}^{\infty}\beta^t V_i\left(p_{ti}\right) + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\sum_{t=0}^{\infty}\beta^t x_t^2\right] \tag{127}$$

Using the insight from before (equation 126) yields the expected welfare loss function.

$$-\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_C C} = \frac{1}{2}\mathbb{E}_0 \left[\frac{\epsilon}{\Theta} \frac{\theta}{(1-\theta)(1-\beta\theta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2 + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) \sum_{t=0}^{\infty} \beta^t x_t^2 \right] \quad (128)$$

$$-\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_C C} = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\epsilon}{\Theta} \frac{\theta}{(1-\theta)(1-\beta\theta)} \pi_t^2 + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) x_t^2 \right] \quad (129)$$

Define $\mathbb{W} \equiv -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_C C}$ as the sum of the discounted current and future consumption loss equivalents as a fraction of steady state consumption (for more details, see below). Finally, the expected welfare loss function can be represented in two ways

$$\mathbb{W} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left(\left(\frac{\epsilon}{\xi}\right) \pi_t^2 + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) x_t^2 \right) \quad (130)$$

$$\mathbb{W}' = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} (\pi_t^2 + \vartheta x_t^2) \quad (131)$$

with $\xi \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$ (as above), $\mathbb{W} = \frac{\epsilon}{\xi}\mathbb{W}'$, and $\vartheta = \frac{\xi}{\epsilon}\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$. $\mathbb{W}'$ is a rescaled welfare loss measure computed with a normalized weight of 1 on the inflation variability.

5.4 Objective Function Under Limited Commitment

This subsection proves that under limited commitment, minimizing the (rescaled) welfare loss function (equation 131) of the representative household amounts to minimizing a central bank objective function of the form as in Debortoli et al. (2014) (cf. equation 141).[26] In particular, it shows that future period losses are discounted by β *times* the probability that the central bank has not deviated from the announced policy path until that period. This is so because the central bank cannot control the expectations about the allocation prevailing off the path on which the agents expect commitments to be honored.

	Description
$W(k_0)$	Planner's value function
Ω^t	Set of possible histories of states without re-opt. up to time t
$\Omega^t_{R,i}$	Set of possible histories of states up to time t, first re-opt. in period i
$x_t(\omega^t)$	Choice variables, depending on the state ω^t
$k_t(\omega^t)$	State variables, depending on the state ω^t
$u(\ldots,\ldots)$	Current period objective function
$p(\omega^t)$	Probability of state ω^t
β	Discount factor

[26] The proof is based on Debortoli and Nunes (2010).

Table 6: Deriving The Central Bank Objective Function: Notation

In our case, the planner's value function is the minimized (rescaled) welfare loss function of the representative household

$$W(k_0) = \min_{\{\pi_t, x_t\}_{t=0}^{\infty}} \underbrace{\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2}(\pi_t^2 + \vartheta x_t^2)}_{\text{(Rescaled) welfare loss function}} \tag{132}$$

(cf. equation 131).[27] To make it comparable to Debortoli and Nunes (2010), use their notation, i.e. let $\frac{1}{2}(\pi_t^2 + \vartheta x_t^2) = u\left(x_t(\omega^t), k_t(\omega^t)\right)$ where $x_t(\omega^t)$ is the control variable depending on the history of events up to period t. (ω^t) and $k_t(\omega^t)$ is the state variable.

$$W(k_0) = \min_{\{x_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(x_t(\omega^t), k_t(\omega^t)\right) \tag{133}$$

The planner's value function can be expressed as

$$\begin{aligned} W(k_0) = \min_{\{x_t\}_{t=0}^{\infty}} \mathbb{E}_0 \Bigg\{ & \sum_{t=0}^{\infty} \sum_{\omega^t \in \Omega^t} \beta^t p(\omega^t) u\left(x_t(\omega^t), k_t(\omega^t)\right) + \\ & \min_{\{x_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \sum_{\omega^t \in \Omega_{R,1}^t} \beta^t p(\omega^t) u\left(x_t(\omega^t), k_t(\omega^t)\right) + \\ & \min_{\{x_t\}_{t=2}^{\infty}} \sum_{t=2}^{\infty} \sum_{\omega^t \in \Omega_{R,2}^t} \beta^t p(\omega^t) u\left(x_t(\omega^t), k_t(\omega^t)\right) + \ldots \Bigg\} \end{aligned} \tag{134}$$

The first line is the path on which commitments are honored in every period. The second line captures each path *starting with* a deviation from the announced policy plan in period 1. The third line captures each path starting with a deviation from the announced policy plan in period 2, and so on. For $\omega^t \in \Omega_{R,i}^i$, with $t \geq i$, $p(\omega^t) = p(\omega^t, \omega_{R,i}^i) = p(\omega^t | \omega_{R,i}^i) p(\omega_{R,i}^i)$.

[27] In Debortoli and Nunes (2010), the value function is termed "objective function".

$$\begin{aligned} W(k_0) = \min_{\{x_t\}_{t=0}^{\infty}} \mathbb{E}_0 \Bigg\{ & \sum_{t=0}^{\infty} \sum_{\omega^t \in \Omega^t} \beta^t p(\omega^t) u\left(x_t(\omega^t), k_t(\omega^t)\right) + \\ & \sum_{i=1}^{\infty} \beta^i p(\omega_{R,i}^i) \left[\sum_{t=i}^{\infty} \sum_{\omega^t \in \Omega_{R,i}^t} \beta^{t-i} p(\omega^t | \omega_{R,i}^i) u\left(x_i(\omega^t), k_i(\omega^t)\right) \right] \Bigg\} \end{aligned} \tag{135}$$

Define $\epsilon_i\left(k_t(\omega_{R,i}^t)\right) \equiv \sum_{t=i}^{\infty} \sum_{\omega^t \in \Omega_{R,i}^t} \beta^{t-i} p(\omega^t | \omega_{R,i}^i) u\left(x_i(\omega^t), k_i(\omega^t)\right)$.

The term $\epsilon_i\left(k_t(\omega_{R,i}^t)\right)$ captures each path starting with a deviation from the announced policy plan in period $i > 0$. Put differently, it captures each path on which the central bank deviates from its policy plan.

$$\begin{aligned} W(k_0) = \min_{\{x_t(\Omega^t)\}_{t=0}^{\infty}} \mathbb{E}_0 \Bigg\{ & \sum_{t=0}^{\infty} \beta^t p(\omega^t) u\left(x_t(\omega^t), k_t(\omega^t)\right) + \\ & \sum_{i=1}^{\infty} \beta^i p(\omega_{R,i}^i) \epsilon_i\left(k_t(\omega_{R,i}^t)\right) \Bigg\} \end{aligned} \tag{136}$$

Let the degree of commitment be time-invariant. Then $p(\omega^t) = \gamma^t$ and $p(\omega_{R,i}^i) = \gamma^{i-1}(1-\gamma)$.

$$\begin{aligned} W(k_0) = \min_{\{x_t(\Omega^t)\}_{t=0}^{\infty}} \mathbb{E}_0 \Bigg\{ & \sum_{t=0}^{\infty} (\beta\gamma)^t u\left(x_t(\omega^t), k_t(\omega^t)\right) + \\ & \beta(1-\gamma)\epsilon_1\left(k_1(\omega_{R,1}^t)\right) + \beta^2\gamma(1-\gamma)\epsilon_2\left(k_2(\omega_{R,2}^t)\right) + \\ & \beta^3\gamma^2(1-\gamma)\epsilon_3\left(k_3(\omega_{R,3}^t)\right) + \dots \Bigg\} \end{aligned} \tag{137}$$

which is equal to

$$W(k_0) = \min_{\{x_t(\Omega^t)\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} (\beta\gamma)^t \left[u\left(x_t(\omega^t), k_t(\omega^t)\right) + \beta(1-\gamma)\epsilon_{t+1} \right] \right\} \tag{138}$$

Expectations about allocations prevailing off the path on which the agents expect commitments to be honored, i.e. $\sum_{t=0}^{\infty} \mathbb{E}_0 \epsilon_{t+1}$, are exogenous to the central bank's optimization problem. Hence

$$W(k_0) = \min_{\{x_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} (\beta\gamma)^t u\left(x_t(\omega^t), k_t(\omega^t)\right) + t.i.p. \right\} \tag{139}$$

$$W(k_0) = \min_{\{\pi_t, x_t\}_{t=0}^{\infty}} \underbrace{\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} (\beta\gamma)^t \frac{1}{2}(\pi_t^2 + \vartheta x_t^2) + t.i.p. \right\}}_{\substack{\text{The central bank objective function} \\ \text{as in Debortoli et al. (2014)}}} \tag{140}$$

where $t.i.p$ denotes terms independent of policy. In sum, we proved that minimizing the (rescaled) welfare loss function (equation 131) of the representative household amounts to minimizing a central bank objective function of the form as in Debortoli et al. (2014) (cf. equation 141).

5.5 Model Solution

This subsection derives the optimality condition of the optimal monetary policy problem under time-varying limited commitment. The derivation is based on the work of Debortoli et al. (2014), Galí (2015), and Woodford (2005). The central bank problem is to minimize the (rescaled) welfare loss function over $\{\pi_t, x_t\}_{t=0}^{\infty}$

$$\min_{\{\pi_t, x_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \prod_{i=0}^{t-1} \gamma_i \frac{1}{2}(\pi_t^2 + \vartheta x_t^2) \tag{141}$$

where γ_t denotes the central bank's credibility in period t and $\prod_{i=0}^{-1} \gamma_i = 1$.[28] The policy problem is subject to the New Keynesian Phillips curve

$$\pi_t = \kappa x_t + \beta\gamma_t \mathbb{E}_t \pi_{t+1} + \beta(1-\gamma_t)\mathbb{E}_t \pi_{t+1}^d + u_t \tag{142}$$

where $\mathbb{E}_t \pi_{t+1}$ is the inflation rate that is expected to prevail if commitments are honored in period $t+1$ and $\mathbb{E}_t \pi_{t+1}^d$ is the inflation rate that is expected to prevail if the central bank deviates from the announced policy path in period $t+1$.

Let us express the optimization problem as a Lagrangian "subject to the constraint that [the optimal evolution of both choice variables from $t = 0$ onward] represents a possible rational expectation equilibrium" (Woodford (2005,

[28] It is γ_0 that is associated to (x_1, π_1) rather than γ_1 because the probability with which the agents expect the period 0 commitment to be implemented in period 1 is governed by the central bank's credibility in period 0. In other words, γ_t has to be interpreted as the probability of seeing future policy being implemented as promised (cf. Galí (2015, 129)).

488)). Assume that the inflation rate which is expected to prevail if the central bank reneges on past policy commitment in $t+1$ is an arbitrary (linear) function of the state variable(s) in $t+1$. Formally, assume $\mathbb{E}_t\pi^d_{t+1} = \mathbb{E}_t f_{t+1}(u_{t+1})$ with the (time-varying) functional form of f_{t+1} unknown.

$$\begin{aligned}\mathcal{L} =& \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \prod_{i=0}^{t-1} \gamma_i \Bigg\{ \frac{1}{2}(\pi_t^2 + \vartheta x_t^2) + \lambda_t \Bigg(\pi_t - \kappa x_t \\ & - \beta\gamma_t \pi_{t+1} + \beta(1-\gamma_t)\pi^d_{t+1} - u_t \Bigg) \Bigg\}\end{aligned} \tag{143}$$

$$\begin{aligned}\mathcal{L} =& \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \prod_{i=0}^{t-1} \gamma_i \Bigg\{ \frac{1}{2}(\pi_t^2 + \vartheta x_t^2) + \lambda_t \Bigg(\pi_t - \kappa x_t \\ & - \beta\gamma_t \pi_{t+1} + \beta(1-\gamma_t) f_{t+1}(u_{t+1}) - u_t \Bigg) \Bigg\}\end{aligned} \tag{144}$$

with λ_t being the Lagrange multiplier associated to the New Keynesian Phillips curve in period t. The first order condition (FOC) with respect to π_t, for each t and every potential state of the world is

$$\beta^t \prod_{i=0}^{t-1} \gamma_i (\pi_t + \lambda_t) - \beta^{t-1} \prod_{i=0}^{t-2} \gamma_i \lambda_{t-1} \beta \gamma_{t-1} = 0 \tag{145}$$

$$\pi_t = \lambda_{t-1} - \lambda_t \tag{146}$$

if $\gamma_i > 0 \ \forall i \in \{0, \dots, t-1\}$. The first order condition (FOC) with respect to x_t, for each t and every potential state of the world is

$$\beta^t \prod_{i=0}^{t-1} \gamma_i (\vartheta x_t - \lambda_t \kappa) = 0 \tag{147}$$

$$\lambda_t = \frac{\vartheta}{\kappa} x_t \tag{148}$$

Combine

$$\pi_t = \frac{\vartheta}{\kappa} x_{t-1} - x_t \tag{149}$$

$$x_t = x_{t-1} - \frac{\kappa}{\vartheta} \pi_t \tag{150}$$

iterate

$$x_t = -\frac{\kappa}{\vartheta}\left[\pi_t + \pi_{t-1} + \ldots + \pi_0\right] + x_{-1} \tag{151}$$

$$x_t = -\frac{\kappa}{\vartheta}\left[\pi_t + \pi_{t-1} + \ldots + \pi_0 - \lambda_{-1}\right] \tag{152}$$

$$x_t = -\frac{\kappa}{\vartheta}\left[\hat{p}_t - \lambda_{-1}\right] \tag{153}$$

where $\hat{p}_t \equiv \pi_t + \hat{p}_{t-1}$ and $\hat{p}_{-1} = 0$ as in Galí (2015, 135) if $\gamma_i > 0 \; \forall i \in \{0, \ldots, t-1\}$.[29] By construction, deviations from the announced policy path in period t imply $\lambda_{t-1} = 0$, as in Debortoli et al. (2014).[30] [31] Consequently, with $t = 0$ being the initial period of the policy plan

$$x_t = -\frac{\kappa}{\vartheta}\hat{p}_t \tag{154}$$

where $\hat{p}_t = \pi_t + \hat{p}_{-1}$ and $\hat{p}_{t-1} = 0$. For $t > 0$, the optimal output gap depends not only on the current inflation rate but also on lagged inflation rates. That is, there is a history dependence in the optimal output gap.

This finding previews the result that under (limited) commitment it is both possible and optimal to commit to future policy responses when facing a current period cost-push shock. The reason is that such a commitment affects the household's expectations which in turn affect current period variables (in particular, the inflation rate). Consequently, less of a current period variability in the output gap is necessary to achieve the optimal inflation rate. This is beneficial because the welfare loss function is strictly convex in the inflation rate and the output gap.[32]

As an intermediary step, re-express the New Keynesian Phillips curve in terms of $\hat{p}_t$. Use the definition of $\hat{p}_t$, and replace x_t with the (combined) first order condition of the optimal monetary policy problem (equation 154).

[29] With $\gamma_0 = 0$, the central bank's first order condition is $x_t = -\frac{\kappa}{\vartheta}\pi_t$ as in Galí (2015, 130). With $\gamma_i = 0$ for $i > 0$, equation 12 applies $\forall t = \{0, \ldots, i-1\}$.

[30] Suppose $\lambda_{-1} = 0$. Then, from equation 146 and 148 it follows that $x_0 = -\frac{\kappa}{\vartheta}\pi_0$ which is the the optimality condition for the period in which the policy plan is first implemented (cf. Galí (2015, 130, 135)). In other words, setting the non-physical $\lambda_{-1} = 0$ is akin to a deviation from the announced policy path in period 0.

[31] In the period of the policy implementation ($t = 0$), the *ratio* between the inflation rate and the output gap is independent of central bank credibilities. However, the *level* of the inflation rate and the output gap changes with $\{\gamma_t\}_{t=0}^{T}$ (cf. equation 173).

[32] A more detailed discussion is provided in Galí (2015, 137).

$$\pi_t = \kappa x_t + \beta\gamma_t \mathbb{E}_t \pi_{t+1} + \beta(1-\gamma_t)\mathbb{E}_t \pi^d_{t+1} + u_t \tag{155}$$

$$\hat{p}_t - \hat{p}_{t-1} = \kappa x_t + \beta\gamma_t \left(\mathbb{E}_t \hat{p}_{t+1} - \hat{p}_t\right) + \beta(1-\gamma_t)\left(\mathbb{E}_t \hat{p}^d_{t+1} - \hat{p}_t\right) + u_t \tag{156}$$

$$\left(\frac{\vartheta(1+\beta)+\kappa^2}{\vartheta}\right)\hat{p}_t = \hat{p}_{t-1} + \beta\gamma_t \mathbb{E}_t \hat{p}_{t+1} + \beta(1-\gamma_t)\mathbb{E}_t \hat{p}^d_{t+1} + u_t \tag{157}$$

Realize that, by definition, $\mathbb{E}_t \hat{p}^d_{t+1} = \mathbb{E}_t \pi^d_{t+1} + \hat{p}_t$.

$$\left(\frac{\vartheta(1+\beta\gamma_t)+\kappa^2}{\vartheta}\right)\hat{p}_t = \hat{p}_{t-1} + \beta\gamma_t \mathbb{E}_t \hat{p}_{t+1} + \beta(1-\gamma_t)\mathbb{E}_t \pi^d_{t+1} + u_t \tag{158}$$

$$\hat{p}_t = \mu_t \left[\hat{p}_{t-1} + \beta\gamma_t \mathbb{E}_t \hat{p}_{t+1} + \beta(1-\gamma_t)\mathbb{E}_t \pi^d_{t+1} + u_t\right] \tag{159}$$

where $\mu_t \equiv \frac{\vartheta}{\vartheta(1+\beta\gamma_t)+\kappa^2}$. Let $u_t \sim AR(1)$ with $\mathbb{E}(\varepsilon^u_t) = 0$ and $V(\varepsilon^u_t) = \sigma^2_{\varepsilon^u_t}$.

$$u_t = \rho_u u_{t-1} + \varepsilon^u_t \tag{160}$$

Apply the method of undetermined coefficients and guess the time-varying solution for $\hat{p}_t$ to be

$$\hat{p}_t = a_t \hat{p}_{t-1} + c_t u_t \tag{161}$$

Further guess the time-varying (linear) solution $\pi^d_t = \hat{h}_t u_t$ with $\hat{h}_t$ unknown. Plug the two guesses into the re-expressed Phillips curve (equation 159).[33]

$$\hat{p}_t = \mu_t \left[\hat{p}_{t-1} + \beta\gamma_t \left[a_{t+1}\hat{p}_t + c_{t+1}\rho_u u_t\right] + \beta(1-\gamma_t)\hat{h}_{t+1}\rho_u u_t + u_t\right] \tag{162}$$

$$\hat{p}_t = \frac{\mu_t}{1-\mu_t\beta\gamma_t a_{t+1}}\left[\hat{p}_{t-1} + (1+\beta\gamma_t c_{t+1}\rho_u + \beta(1-\gamma_t)\hat{h}_{t+1}\rho_u)u_t\right] \tag{163}$$

Solve recursively for $a_t \in (0,1)$.

$$a_t = \frac{\mu_t}{1-\mu_t\beta\gamma_t a_{t+1}} \tag{164}$$

[33] γ_t is *not* a random variable.

Realize that a deviation from the announced policy path in t requires $\hat{p}_{t-1} = 0$ (cf. equation 154). From the guess for $\hat{p}_t$ (equation 161) we get

$$\hat{p}_t^d = a_t \hat{p}_{t-1} + c_t u_t \tag{165}$$
$$\hat{p}_t^d = c_t u_t \tag{166}$$

Furthermore, by definition,

$$\hat{p}_t^d = \pi_t^d + \hat{p}_{t-1} \tag{167}$$
$$\hat{p}_t^d = \pi_t^d \tag{168}$$

Because I assume $\pi_t^d = \hat{h}_t u_t$ it must be that $\hat{h}_t = c_t \ \forall t$. Solve recursively for c_t, $t \in \{0, \dots, T\}$, using $\{a_j\}_{t=0}^{T+1}$ from above.

$$c_t = \frac{\mu_t(1 + \beta\gamma_t c_{t+1}\rho_u + \beta(1-\gamma_t)\hat{h}_{t+1}\rho_u)}{1 - \mu_t\beta\gamma_t a_{t+1}} \tag{169}$$
$$c_t = \frac{\mu_t(1 + \beta c_{t+1}\rho_u)}{1 - \mu_t\beta\gamma_t a_{t+1}} \tag{170}$$

and $c_{T+i} = \frac{\vartheta}{\kappa^2 + \vartheta(1-\beta\rho_u)} \ \forall i \geq 1$ as in Galí (2015, 130).

The optimality condition (equation 154), together with the guess for $\hat{p}_t$ (equation 161) and the solution for the coefficients (in particular, $c_t = \hat{h}_t$) yields the time-varying model solution.

$$\hat{p}_t = a_t \hat{p}_{t-1} + c_t u_t \tag{171}$$
$$x_t = a_t x_{t-1} - \frac{c_t \kappa}{\vartheta} u_t \tag{172}$$
$$x_t = a_t x_{t-1} - \frac{\hat{h}_t \kappa}{\vartheta} u_t \tag{173}$$

where $\mu_t \equiv \frac{\vartheta}{\vartheta(1+\beta\gamma_t)+\kappa^2} \ \forall t$, $a_t = \frac{\mu_t}{1-\mu_t\beta\gamma_t a_{t+1}} \ \forall t$, $\hat{h}_t = \frac{\mu_t(1+\beta\hat{h}_{t+1}\rho_u)}{1-\mu_t\beta\gamma_t a_{t+1}} \ \forall t \in \{0, \dots, T\}$, and $\hat{h}_{T+i} = \frac{\vartheta}{\kappa^2+\vartheta(1-\beta\rho_u)} \ \forall i \geq 1$.

5.6 Proofs

Suppose $t = \{0, 1, 2, 3\}$ with $T = 2$ and assume $x_{-1} = 0$. From the model solution (equation 173) and the combined first order condition of the central bank (equation 150) it follows that

$$x_{t,h} = a_t x_{t-1} - \frac{\hat{h}_t \kappa}{\vartheta} u_t \tag{174}$$

$$\pi_{t,h} = \hat{h}_t u_t + \frac{\vartheta}{\kappa}(1 - a_t) x_{t-1} \tag{175}$$

if past policy commitments are honored (which is indicated by the subscript h). In terms of $\tilde{u} = \{u_t\}_{t=0}^{T+1}$, the optimal $\{x_{t,h}\}_{t=0}^{2}$ are

$$x_{0,h} = -\frac{\kappa}{\vartheta} \hat{h}_0 u_0 \tag{176}$$

$$x_{1,h} = -\frac{\kappa}{\vartheta} \left[\hat{h}_1 u_1 + a_1 \hat{h}_0 u_0\right] \tag{177}$$

$$x_{2,h} = -\frac{\kappa}{\vartheta} \left[\hat{h}_2 u_2 + a_2 \hat{h}_1 u_1 + a_2 a_1 \hat{h}_0 u_0\right] \tag{178}$$

In terms of $\tilde{u}$, the optimal $\{\pi_{t,h}\}_{t=0}^{2}$ are

$$\pi_{0,h} = \hat{h}_0 u_0 \tag{179}$$

$$\pi_{1,h} = \hat{h}_1 u_1 - (1 - a_1) \hat{h}_0 u_0 \tag{180}$$

$$\pi_{2,h} = \hat{h}_2 u_2 - (1 - a_2) \left[\hat{h}_1 u_1 + a_1 \hat{h}_0 u_0\right] \tag{181}$$

As in Debortoli et al. (2014), policy deviations in period s are associated to $x_{s-1} = 0$. Moreover, policy deviations are accompanied by a complete and permanent loss of central bank credibility, i.e. $\gamma_t = 0 \ \forall t \in \{s, ..., 3\}$. In other words, if the central bank deviates from past policy commitment in period s, the optimal $x_{t,d}$ and $\pi_{t,d}$ are, for $t = \{s, ..., 3\}$, equal to the discretionary solution (policy deviations are indicated by the subscript d).[34]

$$x_{t,d} = -\frac{\hat{h}_t \kappa}{\vartheta} u_t \tag{182}$$

$$\pi_{t,d} = \hat{h}_t u_t \tag{183}$$

In period 3, the central bank implements the discretionary solution with certainty.[35] This implies $a_3 = \frac{\vartheta}{\vartheta + \kappa^2}$. Using equation 164, a_2 is given by

[34]Note that $\gamma_t = 0$ does not *per se* imply that the central bank behaves discretionary in period t. *In equilibrium,* however, $\gamma_{t-1} = 0$ always implies a policy deviation in period t. The proof is provided below.

[35]The only *consistent* central bank credibility in T is therefore $\gamma_T = 0$.

$$a_2 = \frac{\mu_2}{1 - \mu_2\beta\gamma_2 a_3} \tag{184}$$

$$a_2 = \frac{\frac{\vartheta}{\vartheta(1+\beta\gamma_2)+\kappa^2}}{1 - \frac{\vartheta\beta\gamma_2}{\vartheta(1+\beta\gamma_2)+\kappa^2}\frac{\vartheta}{\vartheta+\kappa^2}} \tag{185}$$

$$a_2 = \frac{\vartheta\left(\vartheta+\kappa^2\right)}{\left[\vartheta(1+\beta\gamma_2)+\kappa^2\right](\vartheta+\kappa^2) - \vartheta^2\beta\gamma_2} \tag{186}$$

Because $\frac{\partial a_2}{\partial \gamma_2} < 0$, a_2 is maximized at $\gamma_2 = 0$ and minimized at $\gamma_2 = 1$.

$$a_2 = \frac{\vartheta\left(\vartheta+\kappa^2\right)}{\left(\vartheta+\kappa^2\right)^2 + \vartheta\beta\kappa^2\gamma_2} \tag{187}$$

$$\min_{\gamma_2} a_2 = \frac{\vartheta\left(\vartheta+\kappa^2\right)}{\left(\vartheta+\kappa^2\right)^2 + \vartheta\beta\kappa^2} > 0 \tag{188}$$

$$\max_{\gamma_2} a_2 = \frac{\vartheta}{\left(\vartheta+\kappa^2\right)} < 1 \tag{189}$$

i.e. $a_2 \in (0,1)$. Again, resorting to equation 164, a_1 is given by

$$a_1 = \frac{\mu_1}{1 - \mu_1\beta\gamma_1 a_2} \tag{190}$$

$$a_1 = \frac{\frac{\vartheta}{\vartheta(1+\beta\gamma_1)+\kappa^2}}{1 - \frac{\vartheta\beta\gamma_2}{\vartheta(1+\beta\gamma_2)+\kappa^2}\frac{\vartheta(\vartheta+\kappa^2)}{[\vartheta(1+\beta\gamma_2)+\kappa^2](\vartheta+\kappa^2)-\vartheta^2\beta\gamma_2}} \tag{191}$$

$$a_1 = \frac{\vartheta\left\{\left[\vartheta(1+\beta\gamma_2)+\kappa^2\right]\left(\vartheta+\kappa^2\right) - \vartheta^2\beta\gamma_2\right\}}{\left\{\left[\vartheta(1+\beta\gamma_2)+\kappa^2\right](\vartheta+\kappa^2) - \vartheta^2\beta\gamma_2\right\}\left(\vartheta(1+\beta\gamma_1)+\kappa^2\right) - \vartheta^2\beta\gamma_1(\vartheta+\kappa^2)} \tag{192}$$

$$a_1 = \frac{\vartheta\left\{\left[\vartheta(1+\beta\gamma_2)+\kappa^2\right]\left(\vartheta+\kappa^2\right) - \vartheta^2\beta\gamma_2\right\}}{(\vartheta+\kappa^2)\left[(\vartheta(1+\beta\gamma_2)+\kappa^2)\left(\vartheta(1+\beta\gamma_1)+\kappa^2\right) - \vartheta^2\beta(\gamma_2+\gamma_1)\right] - \vartheta^3\beta^2\gamma_2\gamma_1} \tag{193}$$

Because $\mu_1 \in (0,1)$ and $\mu_1\beta\gamma_1 a_2 \in (0,1)$, equation 190 implies $a_1 \in (0,1)$. For the sake of tractability, a_0 is expressed as

$$a_0 = \frac{\mu_0}{1 - \mu_0\beta\gamma_0 a_1} \tag{194}$$

$$a_0 = \frac{\frac{\vartheta}{\vartheta(1+\beta\gamma_0)+\kappa^2}}{1 - \frac{\vartheta\beta\gamma_0}{\vartheta(1+\beta\gamma_0)+\kappa^2}} \tag{195}$$

$$a_0 = \frac{\vartheta}{\left[\vartheta(1+\beta\gamma_0)+\kappa^2\right] - \vartheta\beta\gamma_0 a_1} \tag{196}$$

where $a_0 \in (0,1)$ because $\mu_0 \in (0,1)$ and $\mu_0\beta\gamma_0 a_1 \in (0,1)$,. Finally, $\hat{h}_t = a_t \ \forall t \in \{0,1,2,3\}$ if $\rho_u = 0$.

Claim: Time-invariant limited credibility is inconsistent with strategic policy deviations if $\gamma_T = 0$. **Proof:** Suppose $\gamma_T = 0$, $\gamma_t = 0.5\ \forall t < T$, $V_1^h(u_1^j) > V_1^d(u_1^j)$ for exactly one j, and $V_2^h(u_2^k) > V_2^d(u_2^k)$ for exactly one k. **Logic of the proof:** By contradiction: If $V_1^d(u_1^j) \geq V_1^h(u_1^j)\ \forall j$ or $V_2^d(u_2^k) \geq V_2^h(u_2^k)\ \forall k$ it is proven that $\gamma_t = 0.5\ \forall t < T$ is inconsistent with strategic policy deviations.

Realize that $V_3^i(u_3) = 0$ such that $V_2^i(u_2^k) = U_2^i(u_2^k)$. $V_2^h(u_2^k) > V_2^d(u_2^k)$ is satisfied if and only if

$$\begin{aligned}
-\frac{1}{2}\Bigg[&\left(\hat{h}_2 u_2^k - (1-a_2)\left(\hat{h}_1 u_1^j + a_1\hat{h}_0 u_0\right)\right)^2 \\
&+ \frac{\kappa^2}{\vartheta}\left(\hat{h}_2 u_2^k + a_2\left(\hat{h}_1 u_1^j + a_1\hat{h}_0 u_0\right)\right)^2\Bigg] \\
&> -\frac{1}{2}\hat{h}_{2,d}^2\left((u_2^k)^2 + \frac{\kappa^2}{\vartheta}(u_2^k)^2\right)
\end{aligned} \tag{197}$$

$$\begin{aligned}
-\frac{1}{2}\Bigg[&\left(\hat{h}_2^2 (u_2^k)^2 + \left((1-a_2)\left(\hat{h}_1 u_1^j + a_1\hat{h}_0 u_0\right)\right)^2 - 2\hat{h}_2(1-a_2)u_2^k\left(\hat{h}_1 u_1^j + a_1\hat{h}_0 u_0\right)\right) \\
&+ \frac{\kappa^2}{\vartheta}\left(\hat{h}_2^2 (u_2^k)^2 + \left(a_2\left(\hat{h}_1 u_1^j + a_1\hat{h}_0 u_0\right)\right)^2 + 2\hat{h}_2 a_2 u_2^k\left(\hat{h}_1 u_1^j + a_1\hat{h}_0 u_0\right)\right)\Bigg] \\
&> -\frac{1}{2}\hat{h}_{2,d}^2\left((u_2^k)^2 + \frac{\kappa^2}{\vartheta}(u_2^k)^2\right)
\end{aligned} \tag{198}$$

$$\begin{aligned}
(1-a_2)^2&\left(\hat{h}_1 u_1^j + a_1\hat{h}_0 u_0\right)^2 - 2\hat{h}_2(1-a_2)u_2^k\left(\hat{h}_1 u_1^j + a_1\hat{h}_0 u_0\right) \\
&+ \frac{\kappa^2}{\vartheta}\left(a_2^2\left(\hat{h}_1 u_1^j + a_1\hat{h}_0 u_0\right)^2 + 2\hat{h}_2 a_2 u_2^k\left(\hat{h}_1 u_1^j + a_1\hat{h}_0 u_0\right)\right) \\
&< \left(\hat{h}_{2,d}^2 - \hat{h}_2^2\right)\left((u_2^k)^2 + \frac{\kappa^2}{\vartheta}(u_2^k)^2\right)
\end{aligned} \tag{199}$$

$$\begin{aligned}
\Bigg((1-a_2)^2 &+ \frac{\kappa^2}{\vartheta}a_2^2\Bigg)\left(\hat{h}_1 u_1^j + a_1\hat{h}_0 u_0\right)^2 \\
&- 2\left((1-a_2) - \frac{\kappa^2}{\vartheta}a_2\right)\hat{h}_2 u_2^k\left(\hat{h}_1 u_1^j + a_1\hat{h}_0 u_0\right) \\
&< \left(\hat{h}_{2,d}^2 - \hat{h}_2^2\right)\left((u_2^k)^2 + \frac{\kappa^2}{\vartheta}(u_2^k)^2\right)
\end{aligned} \tag{200}$$

Following Galí (2015, 130), $\hat{h}_{2,d} = \hat{h}_{3,d} = \frac{\vartheta}{\kappa^2 + \vartheta(1-\beta\rho_u)}$ is the impact coefficient under a policy deviation. Using equation 170 and $\hat{h}_t = c_t\ \forall t \in \{0,1,2,3\}$, $\hat{h}_2$ (the impact coefficient under honored commitments) is given by

$$\hat{h}_2(\gamma_2 = 0) = \mu_2(1 + \beta\hat{h}_{3,d}\rho_u) \tag{201}$$

$$\hat{h}_2(\gamma_2 = 0) = \frac{\vartheta}{\vartheta + \kappa^2}\left(1 + \frac{\beta\rho_u\vartheta}{\kappa^2 + \vartheta(1-\beta\rho_u)}\right) \tag{202}$$

$$\hat{h}_2(\gamma_2 = 0) = \frac{\vartheta}{\kappa^2 + \vartheta(1-\beta\rho_u)} \tag{203}$$

Finally, remember that $\gamma_2 = 0$ implies $(1-a_2) - \frac{\kappa^2}{\vartheta} a_2 = 0$. In sum, $V_2^h(u_2^k) > V_2^d(u_2^k)$ is satisfied if and only if

$$(1-a_2)^2 + \frac{\kappa^2}{\vartheta} a_2^2 < 0 \tag{204}$$

Because $\vartheta > 0$, the condition for $V_2^h(u_2^k) > V_2^d(u_2^k)$ is never satisfied. This suggests $\gamma_1 = 0$, contradicting what was supposed to be true (namely that $\gamma_t = 0.5\ \forall t < T$ is consistent with strategic policy deviations). Consequently, we have shown that time-invariant limited credibility in period $t < T$ is inconsistent with strategic policy deviations if $\gamma_T = 0$.

Claim: Time-invariant full credibility is inconsistent with strategic policy deviations if $\gamma_T = 0$. **Proof:** Suppose $\gamma_T = 0$, $\gamma_t = 1\ \forall t < T$, $V_1^h(u_1^j) > V_1^d(u_1^j)\ \forall j$, and $V_2^h(u_2^k) > V_2^d(u_2^k)\ \forall k$. **Logic of the proof:** By contradiction: If $V_1^d(u_1^j) \geq V_1^h(u_1^j)$ for some j or $V_2^d(u_2^k) \geq V_2^h(u_2^k)$ for some k it is proven that $\gamma_t = 1\ \forall t < T$ is inconsistent with strategic policy deviations.

Realize that $V_3^i(u_3) = 0$ such that $V_2^i(u_2^k) = U_2^i(u_2^k)$. $V_2^h(u_2^k) > V_2^d(u_2^k)$ is satisfied if and only if

$$\begin{aligned} -\frac{1}{2}\Bigg[&\left(\hat{h}_2 u_2^k - (1-a_2)\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right)^2 \\ &+ \frac{\kappa^2}{\vartheta}\left(\hat{h}_2 u_2^k + a_2\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right)^2\Bigg] \\ &> -\frac{1}{2}\hat{h}_{2,d}^2\left((u_2^k)^2 + \frac{\kappa^2}{\vartheta}(u_2^k)^2\right) \end{aligned} \tag{205}$$

$$\begin{aligned} -\frac{1}{2}\Bigg[&\left(\hat{h}_2^2 (u_2^k)^2 + \left((1-a_2)\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right)^2 - 2\hat{h}_2(1-a_2)u_2^k\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right) \\ &+ \frac{\kappa^2}{\vartheta}\left(\hat{h}_2^2 (u_2^k)^2 + \left(a_2\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right)^2 + 2\hat{h}_2 a_2 u_2^k\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right)\Bigg] \\ &> -\frac{1}{2}\hat{h}_{2,d}^2\left((u_2^k)^2 + \frac{\kappa^2}{\vartheta}(u_2^k)^2\right) \end{aligned} \tag{206}$$

$$\begin{aligned} (1-a_2)^2&\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)^2 - 2\hat{h}_2(1-a_2)u_2^k\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right) \\ &+ \frac{\kappa^2}{\vartheta}\left(a_2^2\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)^2 + 2\hat{h}_2 a_2 u_2^k\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right) \\ &< \left(\hat{h}_{2,d}^2 - \hat{h}_2^2\right)\left((u_2^k)^2 + \frac{\kappa^2}{\vartheta}(u_2^k)^2\right) \end{aligned} \tag{207}$$

$$\begin{aligned}&\left((1-a_2)^2+\frac{\kappa^2}{\vartheta}a_2^2\right)\left(\hat{h}_1u_1^j+a_1\hat{h}_0u_0\right)^2\\&\quad-2\left((1-a_2)-\frac{\kappa^2}{\vartheta}a_2\right)\hat{h}_2u_2^k\left(\hat{h}_1u_1^j+a_1\hat{h}_0u_0\right)\\&\quad<\left(\hat{h}_{2,d}^2-\hat{h}_2^2\right)\left((u_2^k)^2+\frac{\kappa^2}{\vartheta}(u_2^k)^2\right)\end{aligned} \tag{208}$$

Following Galí (2015, 130), $\hat{h}_{2,d}=\hat{h}_{3,d}=\frac{\vartheta}{\kappa^2+\vartheta(1-\beta\rho_u)}$ is the impact coefficient under a policy deviation. Using equation 170 and $\hat{h}_t=c_t\ \forall t\in\{0,1,2,3\}$, $\hat{h}_2$ (the impact coefficient under honored commitments) is given by

$$\hat{h}_2(\gamma_2=0)=\mu_2(1+\beta\hat{h}_{3,d}\rho_u) \tag{209}$$

$$\hat{h}_2(\gamma_2=0)=\frac{\vartheta}{\vartheta+\kappa^2}\left(1+\frac{\beta\rho_u\vartheta}{\kappa^2+\vartheta(1-\beta\rho_u)}\right) \tag{210}$$

$$\hat{h}_2(\gamma_2=0)=\frac{\vartheta}{\kappa^2+\vartheta(1-\beta\rho_u)} \tag{211}$$

Finally, remember that $\gamma_2=0$ implies $(1-a_2)-\frac{\kappa^2}{\vartheta}a_2=0$. In sum, $V_2^h(u_2^k)>V_2^d(u_2^k)$ is satisfied if and only if

$$(1-a_2)^2+\frac{\kappa^2}{\vartheta}a_2^2<0 \tag{212}$$

Because $\vartheta>0$, the condition for $V_2^h(u_2^k)>V_2^d(u_2^k)$ is never satisfied. This suggests $\gamma_1=0$, contradicting what was supposed to be true (namely that $\gamma_t=1\ \forall t<T$ is consistent with strategic policy deviations). Consequently, we have shown that full credibility in period $t<T$ is inconsistent with strategic policy deviations if $\gamma_T=0$.

Claim: Time-invariant zero credibility is consistent with strategic policy deviations. **Proof:** Suppose $\gamma_t=0\ \forall t\leq T$. Then $V_1^h(u_1^j)\leq V_1^d(u_1^j)\ \forall j$ and $V_2^h(u_2^k)\leq V_2^d(u_2^k)\ \forall k$. **Logic of the proof:** Direct proof: If $V_1^h(u_1^j)\leq V_1^d(u_1^j)\ \forall j$ and $V_2^h(u_2^k)\leq V_2^d(u_2^k)\ \forall k$ it is proven that $\gamma_t=0\ \forall t\leq T$ is consistent with strategic policy deviations.

Realize that $V_3^i(u_3)=0$ such that $V_2^i(u_2^k)=U_2^i(u_2^k)$. $V_2^h(u_2^k)\leq V_2^d(u_2^k)$ is satisfied if and only if

$$\begin{aligned}
&\left((1-a_2)^2+\frac{\kappa^2}{\vartheta}a_2^2\right)\left(\hat{h}_1u_1^j+a_1\hat{h}_0u_0\right)^2 \\
&\qquad -2\left((1-a_2)-\frac{\kappa^2}{\vartheta}a_2\right)\hat{h}_2u_2^k\left(\hat{h}_1u_1^j+a_1\hat{h}_0u_0\right) \\
&\qquad \geq\left(\hat{h}_{2,d}^2-\hat{h}_2^2\right)\left((u_2^k)^2+\frac{\kappa^2}{\vartheta}(u_2^k)^2\right)
\end{aligned} \tag{213}$$

From the previous proof, $\hat{h}_{2,d}=\hat{h}_2(\gamma_2=0)$. Moreover, $\gamma_2=0$ implies $(1-a_2)-\frac{\kappa^2}{\vartheta}a_2=0$. In sum, $V_2^h(u_2^k)\leq V_2^d(u_2^k)$ is satisfied if and only if

$$(1-a_2)^2+\frac{\kappa^2}{\vartheta}a_2^2\geq 0 \tag{214}$$

Because $\vartheta>0$, the condition for $V_2^h(u_2^k)\leq V_2^d(u_2^k)$ $\forall k$ is satisfied. This finding confirms what was supposed to be true (namely $\gamma_1=0$ is consistent with strategic policy deviations). What is left to prove is $V_1^h(u_1^j)\leq V_1^d(u_1^k)$ $\forall j$ if the central bank credibility in period 1 is $\gamma_1=0$.

$$V_1^h(u_1^j)\leq V_1^d(u_1^j) \tag{215}$$

$$U_1^h(u_1^j)+\beta p\left(V_2^d(u_2^k)+V_2^d(u_2^{\bar{k}})\right)\leq U_1^d(u_1^j)+\beta p\left(V_2^d(u_2^k)+V_2^d(u_2^{\bar{k}})\right) \tag{216}$$

$$U_1^h(u_1^j)\leq U_1^d(u_1^j) \tag{217}$$

more explicitly

$$-\frac{1}{2}\left[\left(\hat{h}_1u_1^j-(1-a_1)\hat{h}_0u_0\right)^2+\frac{\kappa^2}{\vartheta}(\hat{h}_1u_1^j+a_1\hat{h}_0u_0)^2\right]\leq-\frac{1}{2}\hat{h}_{1,d}^2\left((u_1^j)^2+\frac{\kappa^2}{\vartheta}(u_1^j)^2\right) \tag{218}$$

Make use of the fact that $\gamma_t=0$ $\forall t\leq T$ implies $\hat{h}_0=\hat{h}_1=\hat{h}_{1,d}$.

$$(1-a_1)^2u_0^2-2(1-a_1)u_1^ju_0+\frac{\kappa^2}{\vartheta}\left(a_1^2u_0^2+2a_1u_1^ju_0\right)\geq 0 \tag{219}$$

$$\left((1-a_1)^2+\frac{\kappa^2}{\vartheta}a_1^2\right)u_0^2-2\left((1-a_1)-\frac{\kappa^2}{\vartheta}a_1\right)u_1^ju_0\geq 0 \tag{220}$$

Remember that $\gamma_1=0$ implies $(1-a_1)-\frac{\kappa^2}{\vartheta}a_1=0$. As a consequence, $V_1^h(u_1^j)\leq V_1^d(u_1^j)$ is satisfied if and only if

$$(1-a_1)^2 + \frac{\kappa^2}{\vartheta} a_1 \geq 0 \tag{221}$$

Because $\vartheta > 0$, the condition for $V_1^h(u_1^j) \leq V_1^d(u_1^j)$ $\forall j$ is satisfied. This finding confirms what was supposed to be true (namely $\gamma_t = 0$ $\forall t \leq T$ is consistent with strategic policy deviations).

Claim: There exists a $\bar{\gamma} \in [0, 1]$ such that time-invariant full credibility is consistent with strategic policy deviations if $\gamma_T \geq \bar{\gamma}$ (assuming that $\rho_u = 0$).[36] [37] **Proof:** Suppose $\gamma_T > 0$, $\gamma_t = 1$ $\forall t < T$ and $\rho_u = 0$. Then $V_1^h(\tilde{u}) > V_1^d(\tilde{u})$ and $V_2^h(\tilde{u}) > V_2^d(\tilde{u})$ for each potential shock sequence $\tilde{u}$ if $\gamma_T \geq \bar{\gamma} \in [0, 1]$. **Logic of the proof:** Direct proof: If $\exists$ $\gamma_T \geq \bar{\gamma} \in [0, 1]$ such that $V_1^h(\tilde{u}) > V_1^d(\tilde{u})$ and $V_2^h(\tilde{u}) > V_2^d(\tilde{u})$ for each $\tilde{u}$, it is proven that $\gamma_t = 1$ $\forall t < T$ is consistent with strategic policy deviations if $\bar{\gamma} \leq \gamma_T$ and $\rho_u = 0$.

Realize that $V_3^i(u_3) = 0$ such that $V_2^i(u_2^k) = U_2^i(u_2^k)$. $V_2^h(u_2^k) > V_2^d(u_2^k)$ is satisfied if and only if

$$\begin{aligned} -\frac{1}{2}\Bigg[&\left(\hat{h}_2 u_2^k - (1-a_2)\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right)^2 \\ &+ \frac{\kappa^2}{\vartheta}\left(\hat{h}_2 u_2^k + a_2\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right)^2 \Bigg] \\ &> -\frac{1}{2}\hat{h}_{2,d}^2\left((u_2^k)^2 + \frac{\kappa^2}{\vartheta}(u_2^k)^2\right) \end{aligned} \tag{222}$$

$$\begin{aligned} -\frac{1}{2}\Bigg[&\left(\hat{h}_2^2 (u_2^k)^2 + \left((1-a_2)\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right)^2 - 2\hat{h}_2(1-a_2)u_2^k\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right) \\ &+ \frac{\kappa^2}{\vartheta}\left(\hat{h}_2^2 (u_2^k)^2 + \left(a_2\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right)^2 + 2\hat{h}_2 a_2 u_2^k\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right) \Bigg] \\ &> -\frac{1}{2}\hat{h}_{2,d}^2\left((u_2^k)^2 + \frac{\kappa^2}{\vartheta}(u_2^k)^2\right) \end{aligned} \tag{223}$$

$$\begin{aligned} (1-a_2)^2 &\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)^2 - 2\hat{h}_2(1-a_2)u_2^k\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right) \\ &+ \frac{\kappa^2}{\vartheta}\left(a_2^2\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)^2 + 2\hat{h}_2 a_2 u_2^k\left(\hat{h}_1 u_1^j + a_1 \hat{h}_0 u_0\right)\right) \\ &< \left(\hat{h}_{2,d}^2 - \hat{h}_2^2\right)\left((u_2^k)^2 + \frac{\kappa^2}{\vartheta}(u_2^k)^2\right) \end{aligned} \tag{224}$$

[36] $\gamma_T > 0$ is *assumed* rather than derived from a consistency requirement. Consistency would require $\gamma_T = 0$ because in period $T+1$, the central bank implements the discretionary solution with certainty.

[37] $\rho_u = 0$ is assumed for simplicity.

$$\begin{aligned}&\left((1-a_2)^2+\frac{\kappa^2}{\vartheta}a_2^2\right)\left(\hat{h}_1u_1^j+a_1\hat{h}_0u_0\right)^2\\&\quad-2\left((1-a_2)-\frac{\kappa^2}{\vartheta}a_2\right)\hat{h}_2u_2^k\left(\hat{h}_1u_1^j+a_1\hat{h}_0u_0\right)\\&\quad<\left(\hat{h}_{2,d}^2-\hat{h}_2^2\right)\left((u_2^k)^2+\frac{\kappa^2}{\vartheta}(u_2^k)^2\right)\end{aligned}\tag{225}$$

Let $\{u_t\}_{t=0}^T = \{H, H, H\}$ and remember that with $\rho_u = 0$, $h_t = a_t$ and $h_{t,d} = a_{t,d}$ $\forall t \leq T$.[38]

$$\begin{aligned}&\left((1-a_2)^2+\frac{\kappa^2}{\vartheta}a_2^2\right)\left(\hat{h}_1+a_1\hat{h}_0\right)^2-2\left((1-a_2)-\frac{\kappa^2}{\vartheta}a_2\right)\hat{h}_2\left(\hat{h}_1+a_1\hat{h}_0\right)\\&\quad<\left(\hat{h}_{2,d}^2-\hat{h}_2^2\right)\left(1+\frac{\kappa^2}{\vartheta}\right)\end{aligned}\tag{226}$$

$$\begin{aligned}&\left((1-a_2)^2+\frac{\kappa^2}{\vartheta}a_2^2\right)a_1^2\left(1+a_0\right)^2-2\left((1-a_2)-\frac{\kappa^2}{\vartheta}a_2\right)a_2a_1\left(1+a_0\right)\\&\quad<\left(a_{2,d}^2-a_2^2\right)\left(\frac{\vartheta+\kappa^2}{\vartheta}\right)\end{aligned}\tag{227}$$

Let $\Sigma \equiv a_1(1-a_0)$ where

$$a_1=\frac{\vartheta\left\{\left[\vartheta(1+\beta\gamma_2)+\kappa^2\right]\left(\vartheta+\kappa^2\right)-\vartheta^2\beta\gamma_2\right\}}{(\vartheta+\kappa^2)\left[(\vartheta(1+\beta\gamma_2)+\kappa^2)\left(\vartheta(1+\beta\gamma_1)+\kappa^2\right)-\vartheta^2\beta(\gamma_2+\gamma_1)\right]-\vartheta^3\beta^2\gamma_2\gamma_1}\tag{228}$$

$$a_0=\frac{\vartheta}{[\vartheta(1+\beta\gamma_0)+\kappa^2]-\vartheta\beta\gamma_0a_1}\tag{229}$$

from equation 193 and 196. Remember that $a_{2,d} = \frac{\vartheta}{\vartheta+\kappa^2}$ because $\gamma_t = 0$ if the central bank deviates from past policy commitments in t.

$$\left((1-a_2)^2+\frac{\kappa^2}{\vartheta}a_2^2\right)\Sigma^2-2\left((1-a_2)-\frac{\kappa^2}{\vartheta}a_2\right)a_2\Sigma<\left(a_{2,d}^2-a_2^2\right)\left(\frac{\vartheta+\kappa^2}{\vartheta}\right)\tag{230}$$

$$\left(1-2a_2+a_2^2+\frac{\kappa^2}{\vartheta}a_2^2\right)\Sigma^2-2a_2\Sigma+2a_2^2\Sigma+2\frac{\kappa^2}{\vartheta}a_2^2\Sigma+a_2^2\frac{\vartheta+\kappa^2}{\vartheta}<\frac{\vartheta}{\vartheta+\kappa^2}\tag{231}$$

$$\Sigma^2-2a_2\Sigma(1+\Sigma)+a_2^2\left(\Sigma^2+\frac{\kappa^2}{\vartheta}\Sigma^2+2\Sigma+2\frac{\kappa^2}{\vartheta}\Sigma+\frac{\vartheta+\kappa^2}{\vartheta}\right)<\frac{\vartheta}{\vartheta+\kappa^2}\tag{232}$$

$$\Sigma^2-2a_2\Sigma(1+\Sigma)+a_2^2\frac{\vartheta+\kappa^2}{\vartheta}\left(\Sigma^2+2\Sigma+1\right)<\frac{\vartheta}{\vartheta+\kappa^2}\tag{233}$$

$$\frac{\vartheta+\kappa^2}{\vartheta}\left(\Sigma+1\right)^2a_2^2-2\Sigma(1+\Sigma)a_2+\left(\Sigma^2-\frac{\vartheta}{\vartheta+\kappa^2}\right)<0\tag{234}$$

[38] The proof is similar for other shock sequences. Specifics are provided below.

For $\{u_t\}_{t=0}^T = \{H, H, L\}$, $\{u_t\}_{t=0}^T = \{H, L, H\}$, and $\{u_t\}_{t=0}^T = \{H, L, L\}$ the conditions are given by

$$\frac{\vartheta+\kappa^2}{\vartheta}\left(\Sigma-1\right)^2 a_2^2 + 2\Sigma(1-\Sigma)a_2 + \left(\Sigma^2 - \frac{\vartheta}{\vartheta+\kappa^2}\right) > 0 \tag{235}$$

$$\frac{\vartheta+\kappa^2}{\vartheta}\left(\Sigma-1\right)^2 a_2^2 + 2\Sigma(1-\Sigma)a_2 + \left(\Sigma^2 - \frac{\vartheta}{\vartheta+\kappa^2}\right) < 0 \tag{236}$$

$$\frac{\vartheta+\kappa^2}{\vartheta}\left(\Sigma+1\right)^2 a_2^2 - 2\Sigma(1+\Sigma)a_2 + \left(\Sigma^2 - \frac{\vartheta}{\vartheta+\kappa^2}\right) > 0 \tag{237}$$

respectively. Let $\phi_1 \equiv \frac{\vartheta+\kappa^2}{\vartheta}\left(\Sigma+1\right)^2$, $\phi_2 \equiv 2\Sigma(1+\Sigma)$ and $\phi_3 \equiv \left(\Sigma^2 - \frac{\vartheta}{\vartheta+\kappa^2}\right)$ depend on the initial guess γ_T^G. The threshold for a_2 (i.e. the $\bar{a}$ which satisfies equation 234 with equality, given γ_T^G) is

$$\bar{a}^1 = \frac{-\phi_2 \pm \sqrt{\phi_2^2 - 4\phi_1\phi_3}}{2\phi_1} \tag{238}$$

Because $\frac{\partial a_2}{\partial \gamma_2} < 0$ and because the condition for $V_2^h(u_2^k) > V_2^d(u_2^k)$ is satisfied if and only if $a_2 < \bar{a}$, there must be a associated threshold on $\bar{\gamma}$ for which $\gamma_2 > \bar{\gamma}$ implies $V_2^h(u_2^k) > V_2^d(u_2^k)$. Re-arrange $\bar{a}^1$ such that

$$\bar{a}^1 = \frac{\vartheta\left(\vartheta+\kappa^2\right)}{\left[\vartheta(1+\beta\bar{\gamma}^1)+\kappa^2\right]\left(\vartheta+\kappa^2\right) - \vartheta^2\beta\bar{\gamma}^1} \tag{239}$$

$$\bar{a}^1 = \frac{\vartheta\left(\vartheta+\kappa^2\right)}{\left(\vartheta+\kappa^2\right)^2 + \vartheta\beta\kappa^2\bar{\gamma}^1} \tag{240}$$

$$\bar{\gamma}^1 = \frac{\left(\vartheta+\kappa^2\right)\left(\vartheta - \left(\vartheta+\kappa^2\right)\bar{a}^1\right)}{\vartheta\beta\kappa^2\bar{a}^1} \tag{241}$$

The coefficients associated to a_2 in equation 234 (in particular: Σ) depend on the initial guess γ_T^G. γ_T^G is hence not necessarily equal to $\bar{\gamma}^1$ (which is found to satisfy equation 234 with equality if Σ is formed with γ_T^G). As a consequence, we have to solve for the fixed point of $\bar{\gamma}$ in equation 234 by continued iterations. Proceed as follows: First, compute the difference between γ_T^G and $\bar{\gamma}^I$ (where $\bar{\gamma}^I$ denotes $\bar{\gamma}$ after I iterations). Second, if the difference between γ_T^G and $\bar{\gamma}^I$ is above some critical value, repeat the computation of $\bar{a}^I$ (and the corresponding $\bar{\gamma}^I$) with γ^{I-1} as an input. Repeat until γ^I is sufficiently close to $\bar{\gamma}^I$ and report $\bar{\gamma} = \bar{\gamma}^I$.

For reasonable parameterization, there is a $\gamma_T \geq \bar{\gamma} \in [0, 1]$ such that $V_2^h(\tilde{u}) > V_2^d(\tilde{u})$ for each potential shock sequence $\tilde{u}$. As a consequence, the only consistent central bank credibility in $T-1$ is $\gamma_{T-1} = 1$. This finding confirms what was supposed to be true (namely $\gamma_t = 1$ $\forall t < T$ is consistent with strategic policy deviations if $\gamma_T \geq \bar{\gamma} \in [0, 1]$ and $\rho_u = 0$).